Developing a Line-of-Sight Based Algorithm for Urban Street Network Generalization

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Abstract

Graphs composed of node and link are the most common way of representing street networks for any analysis beyond pure visual representation. The intuitive primal approach to convert a physical street network into an abstract mathematical graph is mapping street intersections as nodes while mapping street segments between intersections as links connecting nodes in the graph. Correspondingly, a dual approach is indirectly mapping street segments to nodes and intersections to links. Although the dual approach seems less intuitive, the dual representation of a street network it creates doesn't suffer from the same inherent low variance problem in the node's 'degree' as the primal representation does and therefore the dual representation usually exhibits the favorable scale-free and small-world properties of a network. Such characteristics of the dual representation makes it a better candidate for certain analyses where topological distance rather than metric distance plays a more important role.

The process of cartographic generalization is usually necessary before the creation of the dual representation because of street topology. Although there have been many cartographic generalization algorithms, few are tailored to satisfy the need of urban street network analyses based on the dual representation. This paper presents a generalization algorithm focusing specifically on urban street networks that utilizes the accompanying drawing of urban blocks and the concept of convex space, medial axis, and line-of-sight. The algorithm can then be implemented either as a modified v.generalize module with the addition of this new method or a new dedicated spatial network analysis module in GRASS.

1. Introduction

According to the State of World Population report, more than half of the world's population is now living in urban areas and the proportion increases continuously in an unprecedented pace, especially in the developing world (Obaid 2007, p 1). This means the importance of towns and cities, which have always been the center stage of humanity, will only escalate. Regardless its size, one dominant feature of an urban settlement is the extensive street network that provides access to those buildings and other land uses in the settlement. Therefore the representation and further analyses of different aspects of the urban street network has served a key role in understanding the urban settlement in depth. Over the years people have developed many methods to represent the urban street network for various purposes. These methods fall into two general categories, either a visual or an abstract representation of the urban street network.

1.1 Visual representations of urban street network

Because the urban street network functions mainly as the circulation system to various land uses in an urban settlement, the most common form of its representation is a map called street map that shows roads and streets in a district or entire city. A typical street map is usually drawn in a way that makes the street network the most visible entity on the map with the location, shape, and width of streets clearly defined. Whether it is in the forms of hand-drawn sketches, printed paper maps, or electronic images displayed on the screen, street maps have proven to be a great tool for navigating cities, towns, and communities. Being a visual representation of the street network, however, the usefulness of the street map depends entirely on the users' vision to discern and their knowledge and intellect to interpret.

In addition to provide linkage and access to buildings and other land uses in the city, the urban street network also provides much-needed open spaces to the otherwise crowded city packed with buildings. The width of urban streets not only determines the traffic they can carry but also the breathing room between buildings across the street. In other words the urban street network itself is the most important urban open space of the city. To most people it is the passage of the network of streets and squares, i.e. the urban voids, lets them experience the city life. Therefore urban morphologists who study urban forms have developed an approach to represent and analyze the network of streets and other open spaces via a black-and-white image called figure-ground drawing. In contrast to the street map, which regards the street network as a distinct entity with well-defined edges, the figure-ground drawing emphasizes open voids such as streets, squares, and parks implicitly via treating them as the "ground" of the "figure" consisting of buildings and other solid masses (Trancik 1986). In a figure-ground drawing figures representing solid masses are usually colored in black or deeply shaded, leaving the ground white or blank. Such a drawing reveals the urban fabric created by a predominant "field" of solids and voids and enables people to study the relative coverage of the urban settlement (ibid).

1.2 Abstract representations of urban street network

Given the limitation of visual representation, to further analyze the street network beyond pure visual interpretation and solve the problems such as "The Seven Bridges of Königsberg," a notable historical problem in mathematics, people have to find other ways to represent the urban street network that is specifically related to the issue under investigation. For example, in the paper written by the pioneering Swiss mathematicians and physicist Leonhard Euler and published in 1736 discussing finding a route through the city of Königsberg in a way that would cross all seven bridges in turn and no repetition, he discovered that the critical feature of such a route is the sequence of bridges crossed ('Leonhard Euler' 2010; 'Seven Bridges of Königsberg' 2010). Therefore he could reformulate the problem in an extremely simplified format that dealt with only seven bridges connecting four landmasses. Because the choice of route inside each landmass and the exact shape, length, or location of each bridge were both found to be irrelevant, one can use an abstract node to represent the landmass and use an abstract link to represent the bridge without altering the nature of the problem. In the terminology of graph theory, whose development has been attributed to that Euler's paper, the node where links connect is called "vertex," the link between nodes is called "edge," and the resulting mathematical structure is called a graph (ibid).

With such an origination from urban street network, it is not surprising that graphs composed of node and link are the most common way of representing street networks for any analysis beyond pure visual representation. Still, depending on the purpose of analysis, there may be more than one way of using graph to represent a street network. Given that the street network mainly serves as the circulation system of an urban settlement, the majority of analyses on urban street network would likely be related to efficient movement from origin to destination within the settlement that involves route choice not unlike "The Seven Bridges of Königsberg" problem. Therefore an intuitive and more popular approach to convert a physical street network into an abstract mathematical graph is mapping street intersections as nodes while mapping street segments between intersections as links connecting nodes in the graph. Because this approach directly maps geographic entities into graph entities with the same dimension, i.e. zero-dimensional intersections to nodes and one-dimensional streets to links, it is called the primal approach (Porta et al 2006). Nowadays the primal approach is the way street networks are encoded and stored in Geographic Information Systems (or GIS) and other spatial databases.

As mentioned earlier, the street network with connected squares also functions as the indispensible open-space system of an urban settlement. When the subject of analysis focuses on the open spaces that street segments implicitly created, each street segment may become either an origin or a destination while how to choose the route to traverse those open spaces in a particular manner become the main concern. Correspondingly, a dual approach is indirectly mapping street segments to nodes and intersections to links. Although the dual approach seems less intuitive, the dual representation of a street network it creates doesn't suffer from the same inherent low variance problem in the node's "degree" as the primal representation does and therefore the dual representation usually exhibits the favorable scale-free and small-world properties of a network (ibid). Such characteristics of the dual representation makes it a better candidate for certain analyses where topological distance rather than metric distance plays a more important role. Space syntax, a well-known methodology for architecture and urban analyses, is one of the examples that rely on the dual approach.

The process of cartographic generalization is usually necessary before the creation of the abstract representation, especially for the dual approach because of street topology. Although there have been many cartographic generalization algorithms, few are tailored to satisfy the need of urban street network analyses based on the dual representation. Therefore the intent of this study is to develop a dual approach oriented generalization algorithm for abstract street network representation that can eventually be implemented either as a modified v.generalize module with the addition of this new method or a new dedicated spatial network analysis module in GRASS. The following subsections develop a generalization algorithm focusing specifically on urban street networks that utilizes the accompanying drawing of urban blocks and the concept of convex space, medial axis, and line-of-sight. Section 2 first describes various approaches to generate the aforementioned representations of urban street network. Section 3 presents an improved generalization algorithm that transforms medial axes into axial lines. Given the algorithm explained in Section 3, Section 4 works through a few experiments to verify the effectiveness of the new algorithm. Section 5 concludes and points to further work in the future.

2. Generation of urban street network representations

Generation of visual representations of urban street network usually required a large-scale comprehensive land survey map that may include land use, land subdivision, block delineation, street layout, building footprints, and/or building construction to serve as the base map. Historical detailed UK Ordnance Survey Maps and the topography layer of the modern UK Ordnance Survey MasterMap are good examples of such base maps. Because the vast amount of high-resolution aerial photographs and satellite imageries of the entire earth has become readily available through online mapping websites or other services in recent years, it is now possible to use only those digital imageries to extract essential information for urban street network representation. Regardless the source of base maps, however, visual representations of urban street network are normally products of a process called cartographic generalization, which may involve methods such as selection, simplification, combination, smoothing, and enhancement ('Cartographic Generalization' 2010). For example, a thematic street map may be derived from block delineation or street layout in a base map, while a figure-ground drawing may be derived from building footprints from the same base map. For these two particular representations, unless the base map is already in the form of a GIS database with separated layers for block delineation and building footprints, completely automated generalization and production may not be possible and most likely require certain degree of human intervention. Automated generalization of street map and figure-ground drawing from other types of base maps is complex and often requires the full treatment of a comprehensive conceptual model of map generalization such as the one proposed by McMaster & Shea (1992). Given its complexity, the automated generalization of visual representations of urban street networks is beyond the scope of this paper and will not be covered. For the purpose of this paper, it is assumed that suitable street maps and figure-ground drawings are already generated properly and available for immediate use. Therefore the remaining part of the section will discuss generating abstract representations of urban street network only.

2.1 Medial Axis

The planning, design, and construction of roads in modern days starts with the delineation of horizontal road alignment, which consists of only straight tangent lines and curves (Strom et al 2009). Horizontal road alignment also closely associates with the center-line surface marking on the finished road. Thus the most straightforward method of generating an abstract primal representation of urban street network is to retrieve the road centerlines. Since it has been the standard practice in GIS to encode transportation networks via their centerlines using the primal approach, one can easily extract the road centerlines from an appropriate layer if she obtains the digital map from a mapping agency. For instance, the UK Ordnance Survey offers a separate Integrated Transportation

Network Layer product within its OS MasterMap geo-database product line that covers the Great Britain. Even if there is no readily available road centerline information to convert to the abstract primal representation of urban street network, one can still generate the road centerlines from scratch using the street map to serve as the boundary of the shape and following certain algorithms to find the equivalent of centerlines of the street network called medial axes in the map.

Blum (1967) coined the term "medial axis" to refer to the symmetric axis or the topological skeleton of an arbitrary shape. Because by definition a point on the medial axis means it must have more than one closest point on the boundary, the mathematical definition of the medial axis of a plane shape is the locus of the centers of circles that are completely contained within and tangent to the boundary of the shape in at least two points ('Medial Axis' 2010). Originally developed to transform a two-dimensional shape, especially that of an organism, into a one-dimensional structure (ibid), the medial axis has found broad applications in pattern recognition, shape analysis (Chin et al 1999), and even other fields as diverse as optimal routing of sensor network in geography and molecular design in chemistry (Leymarie & Kimia 2006).

In his original paper, Blum (1967) used different visual analogies, such as grass fires or wave fronts propagating inward or contour lines moving upward from the boundary to vividly convey the concept of medial-axis computation. Since then many algorithms for medial-axis computation have been proposed. Foskey et al (2003) classified those algorithms into four general categories: thinning algorithms, distance field based algorithms, algebraic methods, and surface-sampling approaches. The latter two follow the formal mathematical definition and rely on the equidistance property of medial axis, and therefore more suitable to be executed in the vector-based implementation. On the other hand, the former two follow Blum's visual analogies and produce the medial axis through a process that shrink the boundary in a constant pace, and therefore are more suitable to be executed in the raster-based implementation. Both thinning function and distance field based function to calculate a cost surface have been available at the outset of raster-based GIS and are commonly found in a modern full-fledged GIS, such as the proprietary ArcGIS or the open-sourced GRASS GIS. Consequently, using a capable GIS may be the most convenient way to generate medial axes on an ad hoc basis. All cases of medial axes shown in this paper are generated through a raster command called r.cost in GRASS, which performs the same function as the aforementioned distance field based algorithms.

One well-known "feature" of the medial axis is its sensitivity to the object's shape, meaning that even a minor perturbation in the boundary may cause spurious deviation on the path of the medial axis (Foskey et al 2003; Jiang & Liu 2010; Tam & Heidrich 2003). Although this means the medial axis can accurately reflect the shape of an object, this also means the medial axis is unstable and thus undesirable as a tool for shape analysis in that it may carry plenty of noises. Many methods have been proposed to "regularize" the medial axis by pruning away insignificant bumps, branches, or other artifacts on the medial axis but still preserving its topology (Tam & Heidrich 2003).

Other issues arise if we directly convert the medial axis into a graph using the primal approach without any generalization and analyze the resulting graph using the centrality indices defined by Freeman (1977, 1979) and others. First of all, if the construction of the primal graph follows the road-centerline-between-nodes rule (Porta et al 2006, pp 711-712) that adds intermediate nodes to indicate linear discontinuity of curve or angled road segments, the graph will be fragmented because the topological distance, i.e. the number of links, between nodes may increase drastically. Even if we disregard the significant characteristics of street geometry by removing those intermediate nodes that have only two links connect to each, the degree, which is the number of links incident upon a node ('Centrality' 2010), of almost all nodes will still fall between 3 and 6 given the number of streets per intersection in a real urban settlement (Porta et al 2006, p 718). As a result, the primal graph of urban street network would normally be neither a small-world network nor a scale-free network, both of which requires high "degree" values ('Small-world Network' 2010; 'Scale-free Network' 2010).

2.2 Axial Line

When we convert the medial axis into a dual graph with only original intermediate nodes removed, the degree does increases but not much, typically from the value of n to 2*n-2. Therefore the characteristics of the resulting graph change little. It is only when appropriate generalization is applied to the medial axis and the generalized medial axis is then converted into a dual graph that could radically alter the characteristics of the graph so it may become a small-world network (Jiang & Claramunt 2004). Many generalization models for urban street network base on the principle of continuity have been proposed (Porta et al 2006, pp 713-714). Given the well-known cognitive property of human wayfinding to go straight at intersections (Dalton, 2001; Dalton 2003; Dalton et al, 2003), the linearity of the street spaces is the more intuitively acceptable principle of continuity to merge single street segments into longer `strokes' (Thomson 2004). The axial lines defined by Hillier and Hanson (1984) and used extensively by the space syntax community are essentially an embodiment of such "strokes."

Space syntax is a set of theories and techniques for the analysis of spatial configurations of all kinds, but it is most popular among architectural and urban researchers as it is in buildings and cities where spatial configuration seems to be a significant aspect of human affairs (Space Syntax Laboratory 2008; 'Space Syntax' 2010). The syntax approach to the analysis of the seemingly continuous and free-flowing urban open spaces "was first to take the predominantly linear nature of urban space seriously, and propose a representation of the street network based on the longest and fewest lines that could be drawn through the system" (Hillier & Vaughan 2007). These lines, which are termed axial lines, are then treated as the nodes of a graph while the junctions are turned into links. Various calculations measuring different characteristics of the graph have been proposed; several measures can find their equivalents in centrality measures of the network theory. For example, the space syntax measures of control, integration, and choice are analogous to the degree (Freeman 1979; Nieminen 1974), closeness (Sabidussi 1966), and betweenness (Freeman 1977), respectively (Hillier & Iida 2005; Porta et al 2006). In short, the adoption of the linear continuity principle and the dual graph approach enables the axial lines in the space syntax analysis to capture key features of the geometry of the street network through the mathematical graph (Hillier & Vaughan 2007).

The problem with axial lines is that the definition of them and the procedure to generate them have not been formulated well enough to prevent any ambiguity (Batty & Rana 2004). According to Hillier and Hanson (1984, p 92), an axial map is “the least set of such straight lines which passes through each convex space and makes all axial links.” And the procedure to generate an axial map is: “first finding the longest straight line that can be drawn ..., then the second longest, and so on until all convex spaces are crossed and all axial lines that can be linked to other axial lines without repetition are so linked” (ibid, p 99). With such vague guidance, it is not surprising that individual users’ intuition and graphic dexterity would determine the outcome, which in turn makes people suspect that each example cannot be replicated by a different user in a different time at a different place (Batty & Rana 2004). Because much is left at the user’s discretion, Desyllas and Duxbury (2001, p 27.6) argue that “the ... axial map cannot provide researchers with reliable and comparable results.” Peponis et al (1998, p 560) thus point out that objectivity in the process of generating axial lines can only arise “from the rigor and repeatability of the procedures used to generate them.”

There have been many attempts to clarify the definition of axial lines and add objectivity into the procedure of generating axial lines and axial maps so that unique and reproducible or even automatic results can be made (Peponis et al 1998; Penn et al 1997; Batty & Rana 2004; Turner et al 2005; Jiang & Liu 2010). These attempts fall into two categories. The first category is an exhaustive approach pioneered by Penn and colleagues that initially creates a drawing called all-lines map showing all possible lines by linking “vertices defining differences in orientation between faces as well as all extensions of faces to meet other faces” (Batty & Rana 2004). In the subsequent steps, certain criteria are applied to remove unqualified axial lines until the least set of the longest straight lines has been identified. The second category is a visibility-based approach pioneered by Batty and colleagues that by slightly altering the definition of axial lines from lines of unobstructed movement to lines of sight. By so doing the viewshed analysis, a common function in raster-based GIS, can be applied to find the viewshed, or variously called visual field or isovist, of each individual locations. Then in an iterative process imitating the aforementioned original Hillier and Hanson procedure, the viewshed with maximum diametric length is selected, the longest axial line is derived from, and that viewshed is removed from the remaining search space until the search space is empty (ibid). Beyond the inherent issues pertinent to each approach as identified in the relevant papers (Batty & Rana 2004; Jiang & Liu 2010; Turner et al 2005), one common problem of these approaches is the counter-intuitive axial lines drawn to pass those barely visible openings between buildings as a result of pursuing superfluously “longest” lines according to the original guidance given by Hillier and Hanson (1984). Those relatively narrow opening are not unlikely to be noticed or walked through by most people in real world. Their emergences obviously depend on the resolution of the source drawing and therefore outcomes of those approaches are scale-dependent. In other words, source drawings of different resolutions for the same site would probably generate different sets of axial lines, which lead the outcomes of these approaches suffer the similar shortcomings to the original hand-drawn guidance — unstable and unreliable.

3. A generalization algorithm that transforms medial axes into axial

lines

3.1 Urban open spaces as a system of beady strings

Since the space syntax is purposed to be a tool for configurational analyses of space, the so-called space is the basic element of the theory upon which control, integration, choice, and other metrics are measured. Consequently the definition of space plays a vital role in the development of the theory. In original development of the theory, Hillier and Hanson (1984, p 90) analogized the continuous open space of an exemplary settlement to “a beady ring system, in that everywhere space widens to form irregular beads, and narrows to form strings, at the same time joining back to itself so that there are always choices of routes from any space to any other space.” At the time they devised the axial lines to represent the straight segments of unobstructed movement and analogized them to the strings of the beady rings, they also identified those fully convex areas and likened those convex spaces to the beads of the rings (ibid, p 91). Accordingly they also devised a complementary graphical tool to the axial map called convex map to study the “stringiness” as well as the “beadiness,” i.e. the one-dimensional and two-dimensional characteristics, of the “rings.” Rooted in architecture and urban design, the space syntax theory defines the convex space as the area circumscribed by view-blocking objects such as wall or other imaginary boundaries connected by edges of those objects such as corners of buildings. The mathematical nature of convex space is in which all points can see all others. In real world this means a place where social behaviors such as interaction among people are more likely to happen. Hillier and Hanson (1984, p 92) define a convex map as: ``the least set of fattest spaces that covers the system," and suggest an algorithm for manually constructing such a convex map: “[s]imply find the largest convex space and draw it in, then the next largest, and so on until all the space is accounted for” (ibid, p 98). With the convex map in place, one can now comprehend the axial map as a representation of the most likely movement pattern in the study region that identifies the “longest and fewest” set of lines stringing various connected convex “beads” together. Collectively those lines form the skeleton of the (open) space of a study region (Jiang & Liu, 2010). From this point of view, it can also be thought what individual axial lines identified are groups of convex spaces in which people can make unobstructed movement easily.

The problem with convex map is it shares the same symptom of axial map: lacking rigorous definition and leaving much at the discretion of users. Both Peponis et al (1997) and Batty and Rana (2004) demonstrate that without further constraints there will be no unique solution to the partitioning of the study region into a minimum number set of convex subspaces. People who are familiar with the concept of spatial data analysis on area data (Haining 2004) may suggest that we can collapse all convex spaces into individual nodes at their centroids and connects those nodes with links according to permeability of original convex spaces to generate a graph. However, without the grouping scheme provided through the identification of axial lines, the convex-map-transformed graph of urban open space also suffers the same problem facing the primal graph that is directly converted from medial axes without any generalization. It might be for this reason the convex map provides no substantial advantage over the axial map and therefore is mainly applied to the analysis of building (Hillier 1996) where wall-separated rooms can be recognized as convex spaces more easily.

3.2 An overview of the algorithm

Clearly what lacks in the space syntax theory are new rules or "constraints" to reasonably and stably partition the continuous urban and/or open space into convex subspaces. Take the beady string analogy again. Imagine what we would do if we want to make one from scratch. We would most likely start with piercing each bead through its center, or at least as closely as possible, and then use a string to thread those perforated beads together in sequence. If intuitively piercing the center of a bead is the most physically secure way to thread the bead, it would not be overly hypothetical to assert that using the medial axis as the guide to assist partitioning of convex spaces could be the most reasonable and stable solution to the problem. The following contents show that the solution is not only feasible but also leads to an improved solution of generating axial lines free from aforementioned problems and quite capable of being automated.

To start with, we use the same T-shape-inside-square figure-ground drawing frequently used in the space syntax literature to illustrate the basic concept and procedure. We use a series of diagrams to demonstrate how medial axes can guide the partitioning of convex spaces and eventually generate the desire axial lines and dual graph. Given the drawing shown in Figure 1(a), the first step is to generate the medial axis of the open space through an appropriate mean. Figure 1(b) shows the desired medial axis on top of the background image, which is a cost surface map generated in GRASS GIS. The ridges in the image are selectively traced to create the medial axis with only the main trunk but no branches.

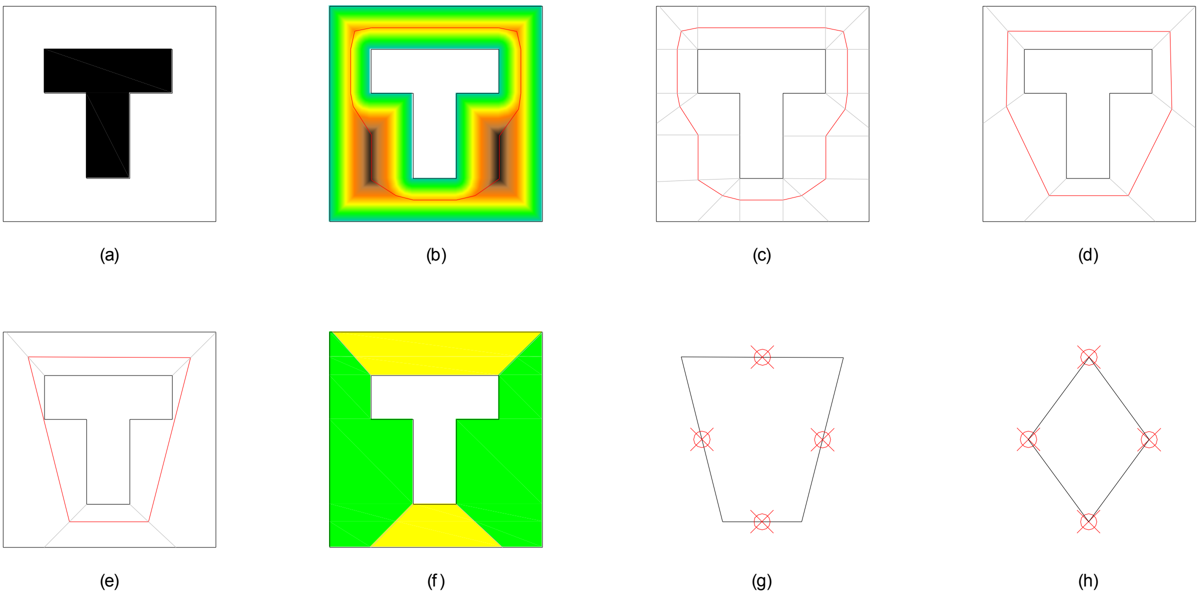

Figure 1. (a) The figure-ground drawing. (b) The traced medial axis on top of the distance field map using shades of color to indicate the distance from boundaries. (c) The initially partitioned convex subspaces. (d) The merged convex spaces. (e) The final set of generalized medial axes, that is, axial lines on top of associated convex-space sets. (f) Convex-space sets colored to differentiate. (g) The axial lines with their midpoints. (h) The final dual graph.

Figure 1(c) shows how the space is partition into convex subspaces so that there is only one straight segment of the medial axis inside each partitioned convex subspace. Theoretically there are two schemes to divide urban open space into convex subspaces based on medial axis: "link-and-joint" spaces versus "all-link" spaces (Figure 2). To some extent, these two schemes can be analogized to the relationship of primal graph versus dual graph, or that of Voronoi tessellation versus Delaunay triangulation. The algorithm follows the all-link scheme to generate the initial set of convex subspaces. Obviously this is not "the least set of fattest spaces that covers the system" yet, therefore a generalization scheme is needed to merge qualified convex spaces into a larger one.

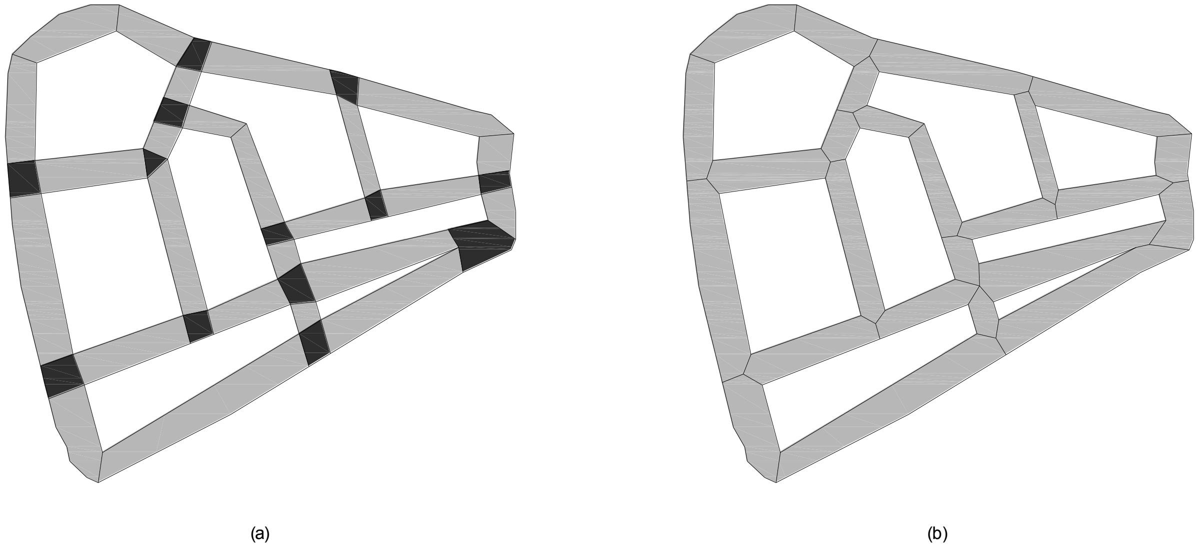

Figure 2. (a) An instance of the link-and-joint convex space partition. (b) An instance of the all-link convex space partition.

Again, the clue is in the medial axis. The generalization follows the aforementioned principle of continuity used in the ICN generalization model (Porta et al 2006, p 713-714) and starts at the first pair of contiguous convex spaces whose internal medial-axis segments form the largest convex angle. That is, start with the convex space pair within which people will experience the least angular change when they move along the medial axis from one segment to the other, to consider whether or not they are qualified for merge. Although more rigorous verification or mathematical proof is still needed, this heuristic prioritization strategy has helped to generate an optimal set of longest, in terms of their shape, and fewest convex spaces in an efficient way in the limited cases tested so far. If the two contiguous convex spaces can be merged to form a new convex space, then the merger proceeds. After the merger, a new line connecting the other two opposite endpoints of the two original joined medial-axis segments is created to replace them and becomes the now generalized medial axis of the new convex space from the merger. This simple generalization approach guarantees that the newly generalized medial-axis segment still connects with other segments so further generalization can proceed. The conditional testing goes on to the next pair of contiguous convex spaces having the least angular change in their medial-axis segments iteratively until no more such pair can be merged to form a new convex space. At this stage, the convex map consists of those merged convex spaces seems to have reasonably achieved “the least set of fattest spaces that covers the system” condition. Figure 1(d) shows the resulting convex map and generalized medial axes. Even though “the criterion for what is a ‘fat’ convex space is never defined” in the definition of convex map (Batty & Rana 2004, p 617), the shapes of the same number of six partitioned convex spaces in Figure 1(d) on average seem to be “rounder” than those shown in Figure 1(a) of the paper by Batty and Rana (2004). But only applying shape metrics (McGarigal et al 2002), such as perimeter-area ratio or linearity index, to compare the overall or average fatness or slimness of the convex spaces between these two figures can objectively accept or reject such superficial claim.

The final phase is the process of merging single street segments into longer “strokes” (Thomson 2004) “until all convex spaces are crossed and all axial lines that can be linked to other axial lines without repetition are so linked” (Hillier & Hanson 1984, p 99). In this phase the same principle of continuity applies so the process begins with the first pair of contiguous convex spaces whose (generalized) medial-axis segments form the largest convex angle. However, since it is no longer possible to create a new convex space by merging, in order to substitute for the convexity requirement we must set a new condition to test if two convex spaces can be merged to form a new non-convex space. Now, what being created by merging two convex spaces together is only a space enclosed by an irregularly shaped polygon. Therefore such a space formed by a group of convex subspaces is termed convex-space set. Given the selected pair of convex spaces, a line connecting the other two opposite endpoints of the two joined medial-axis segments must fall completely inside the area covered by the two contiguous convex spaces in order to guarantee that the newly created line can be qualified as a generalized medial-axis segment and that line still connects with other segments so further generalization can proceed. If this condition is met, then the merger proceeds. And the just-created connecting line becomes the generalized medial axis of the new convex-space set from the merger.

With the two originally joined medial-axis segments, this generalized medial axis creates a triangle that is also completely contained within the new convex-space set. This requirement guarantees unobstructed movement or line of sight, i.e. people can easily see each other, at least within this minimal triangular area along the full longitudinal extent of the new convex-space set. Alternatively, suppose the midpoint of the (generalized) medial-axis segment roughly coincides with the centroid of the convex space or convex-space set. This requirement also means the subspaces inside a convex-space set are linked together by their medial-axis midpoints or even centroids. It thus avoids the aforementioned peculiarly long axial line problem occurs in other axial line generation approaches. The conditional testing goes on to the next pair of contiguous convex spaces and/or convex-space sets having the least angular change in their generalized medial-axis segments iteratively until no more such pair can be merged to form a qualified new convex-space set. At this stage, the definition given by Hillier and Hanson (1984, p 92): “the least set of such straight lines which passes through each convex space and makes all axial links” has been met. Figure 1(e) shows the generalized medial axis as axial lines and underlying convex-space sets;

Figure 1(f) through 1(h) show a map of colored convex-space sets, the axial map with midpoints of axial lines, and a dual graph of the axial map, respectively.

So far we have only dealt with the situation where a pair of adjacent convex spaces and/or convex-space sets meets, i.e. two (generalized) medial-axis segments meet. What if three or more medial-axis segments meet? To handle the situation of more than two medial-axis segments meet, we have to specify the rules of merging for the following three possible ways that a medial-axis segment can interact with others. With these rules, all merging situations can be appropriately managed.

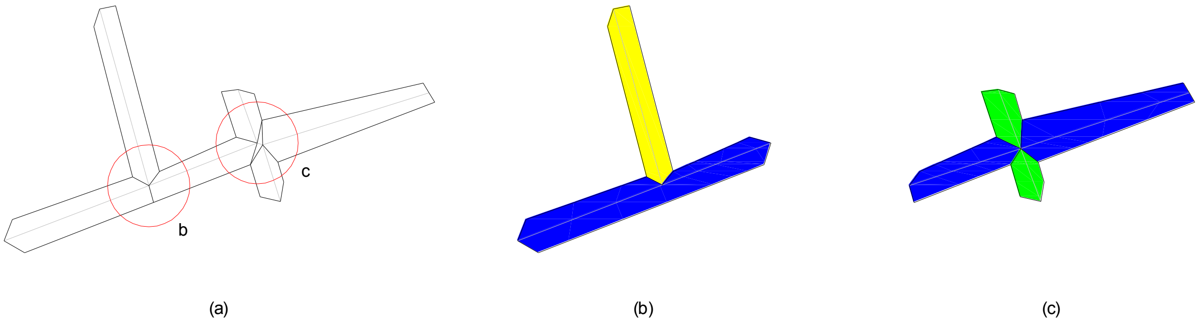

Figure 3. (a) "Join with" situation: three-way (left circle) and four-way (right circle). (b) "Touch upon" situation. (c) "Cross over" situation.

a) Join with (Figure 3(a)): The typical situation happens at a node in a graph where two or more links meet. All merging situations start this way, no matter how many links join at one node. Since this is the situation we have been dealing with so far, simply follow the existing rule outline above.

b) Touch upon (Figure 3(b)): Once qualified convex spaces are merged into a convex-space set (e.g. the blue street), those remaining convex spaces (e.g. the yellow street) sharing common borders with the new convex-space set on only one side are in this situation. This happens only after the first iteration of convex-space sets is created. The merging process will create a new generalized medial axis segment for the merged "trunk" convex-space set. Consequently other "branching" convex spaces or convex-space sets touched upon it have to extend or trim their (generalized) medial axis segments and adjust their associated borders to once again touch upon the new trunk medial axis on only one side. Although the "trunk" cannot further merge with its "branches," merging between those "branches" might be possible. This rule handles the typical road junction with three arms forming either a T or Y shape in the real world.

c) Cross over (Figure 3(c)): This situation also happens after the first iteration of convex-space sets is created. There may be two branches (e.g. the green street) touching upon the trunk (e.g. the blue street) on the opposite sides that are also qualified to be merged together. The qualification is similar but slightly modified. Because the (generalized) medial-axis segments of two crossing branches may be parallel but offset a bit so not intersect inside their combined boundaries, the qualification for merger now relies on the connection of both midpoints and endpoints of the merging medial-axis segments rather than the original rule of forming a triangle that falls completely inside the combined boundaries. Figure 4 shows an example of such a situation. Again the condition is set to prevent the aforementioned peculiarly long axial line problem occurs in other axial line generation approaches.

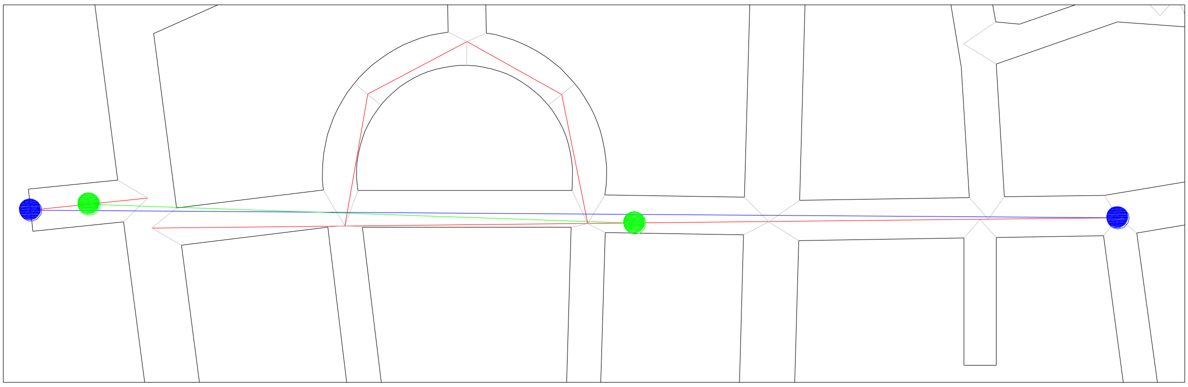

Figure 4. The blue line connecting the two endpoints of the red (generalized) medial axes is completely within the boundaries of the streets, while the green line connecting the two midpoints of the red (generalized) medial axes cuts through an almost invisible portion of the corner. This figure also shows how a curved street section should be subdivided to create generalized linear medial-axis segments.

The revised merging process for the cross-over situation then works as follows. Suppose the (shared) borders connecting to vertices where two generalized medial-axis segments touch on them were removed. If the two new lines, one connecting the two opposite endpoints and the other connecting the midpoints of the two facing (and generalized) medial-axis segments, both fall inside the boundaries of the branches and the trunk, then the removal of the medial-axis vertex is permanent, otherwise the removal is reversed. When the merging proceeds, the just-created endpoint-connecting line becomes the generalized medial axis of the new convex-space set from the merger. And a new trunk crossing over the old trunk is formed. Of course, because the new crossing generalized medial-axis segment will intersect with the crossed medial-axis segment at different location, associated vertices and borders of both trunks have to be adjusted accordingly. This rule takes care of the most common 4-way road intersection in the real world. This and previous rules can be combined to iteratively deal with the 5-way or even 6-way road interaction in the real world.

4. Verification of the algorithm

4.1 Experiments with a set of urban environments

This subsection describes the result of putting the algorithm through the same set of urban environments used by Jiang and Liu (2010) in order to verify the effectiveness of the algorithm. Please refer to Figure 5 of their paper for comparison. Figure 5 shows the testing result of typical street patterns of three, four, and eight blocks (Jacobs 1995). The figure shows that the resulting axial lines still closely resemble the shape of medial axes while significantly reduce the number of constituent segments. By not pursuing absolute fewest and longest axial lines, the proposed algorithm becomes an effective way to generalize the medial axis.

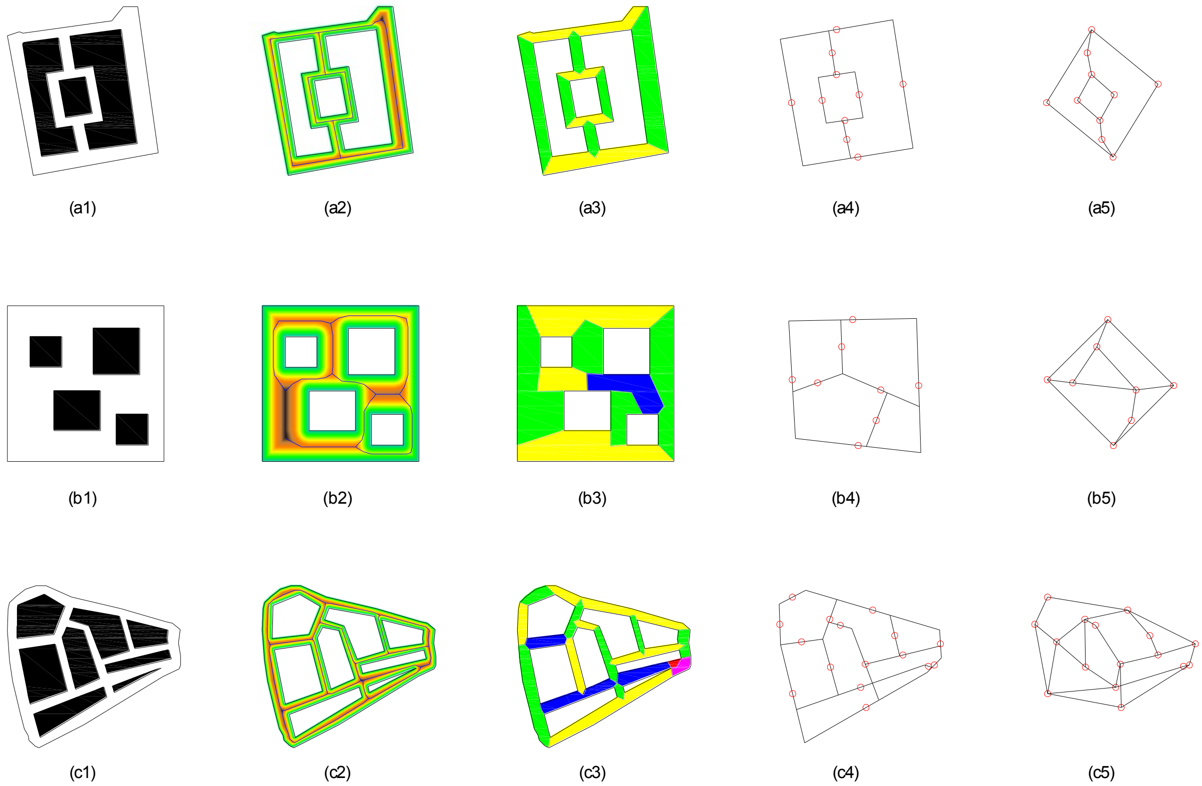

Figure 5. Row (a), (b), and (c) is the result of the three-, four-, and eight-block tests. Column (1) shows figure-ground drawings; column (2) shows traced medial axes on top of the distance field map; column (3) shows the final convex-space sets colored to differentiate; column (4) shows the final axial lines with their midpoints; column (5) shows the final dual graph.

Figure 6 shows the result of testing the algorithm with a real urban street pattern in Barnsbury, London, UK around the Barnard Park. This case demonstrates how curved sections of streets should be converted into straight segments. The medial axis of a curved section of a street is obviously, a curve. Curves are incompatible with the theoretical foundation of linearity or line-of-sight in the algorithm and therefore must be generalized into a series of linear segments. The rule for curved street section applies the same principles that have been followed from the outset. First, the algorithm intends to create reasonable convex spaces instead of absolutely fewest and longest axial lines. Second, the generalized medial-axis segment must fall completely inside the partitioned convex subspace. Therefore the rule is simply dividing the curve into the least number of equi-length sections in which the line connecting the two ends of the curved medial-axis segment must fall completely inside the section in order to become the generalized linear medial-axis segment. Also note that in this case the rule of crossing over has prevented the further merging of at least one pair of crossing convex space and convex-space set because the line connecting midpoints of their (generalized) medial-axis segments partly falls outside the boundaries, even though the line connecting the two far endpoints of their medial-axis segments does fall inside the boundaries. Please refer back to Figure 4 to see an enlarged portion of Figure 6 for illustration. This example once again shows that the algorithm does not intend to create the absolutely longest axial line by joining apparently discontinued convex-space sets together.

Figure 6. (a) The traced medial axes on top of the distance field map. (b) The final convex-space sets colored to differentiate. (c) The final axial lines with their midpoints.

Figure 7 shows some important characteristics of the algorithm. This case demonstrates that the algorithm is quite stable in that it can tolerate both the distortion of the original shape (Figure 7(a)) and imprecision of tracing the medial axis (Figure 7(b)), and still generates similar results. However, during the initial convex-space partitioning phase, the rule of partitioning into fattest convex subspaces must be followed. That is, whenever there is more than one way to partition the convex subspaces, the rule of thumb is to partition them in a way that will create “fattest” convex subspaces. That is, the overall diameters of circles that circumscribe the resulting convex space must be smallest. Figure 7(c) illustrates what might happen if the rule is not followed.

Figure 8 demonstrates that the algorithm can also handle a building plan as complex as the National Gallery, London, UK. Finally, the Figure 9 shows the result of applying the algorithm to the same French town Gassin used by Hiller and Hanson (1984) and Batty and Rana (2004) for comparison.

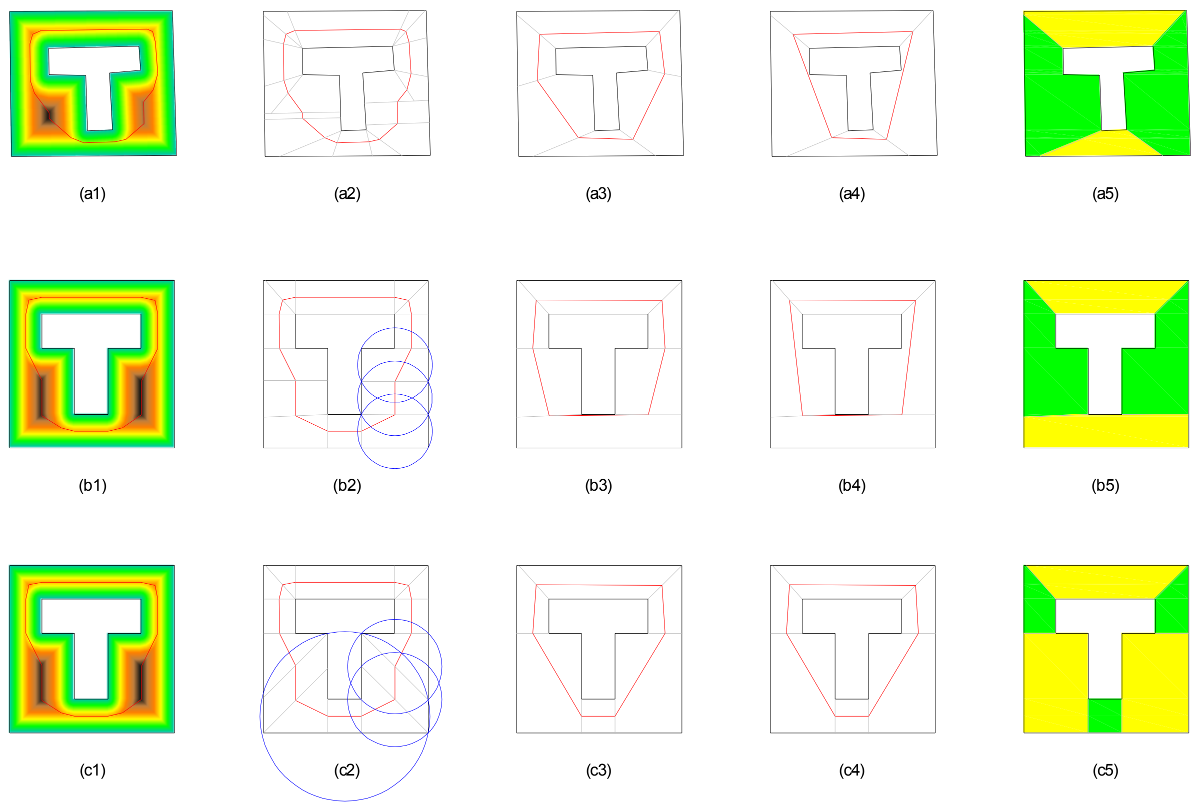

Figure 7. Row (a), (b), and (c) is the testing result of the shape distortion, imprecise tracing, and slim partitioning cases, respectively. Column (1) shows traced medial axes on top of the distance field map; column (2) shows the initially partitioned convex subspaces with some of the circumscribed circles colored in blue; column (3) shows the merged convex spaces; column (4) shows the final set of generalized medial axes, that is, axial lines on top of associated convex-space sets; column (5) shows the final convex-space sets colored to differentiate.

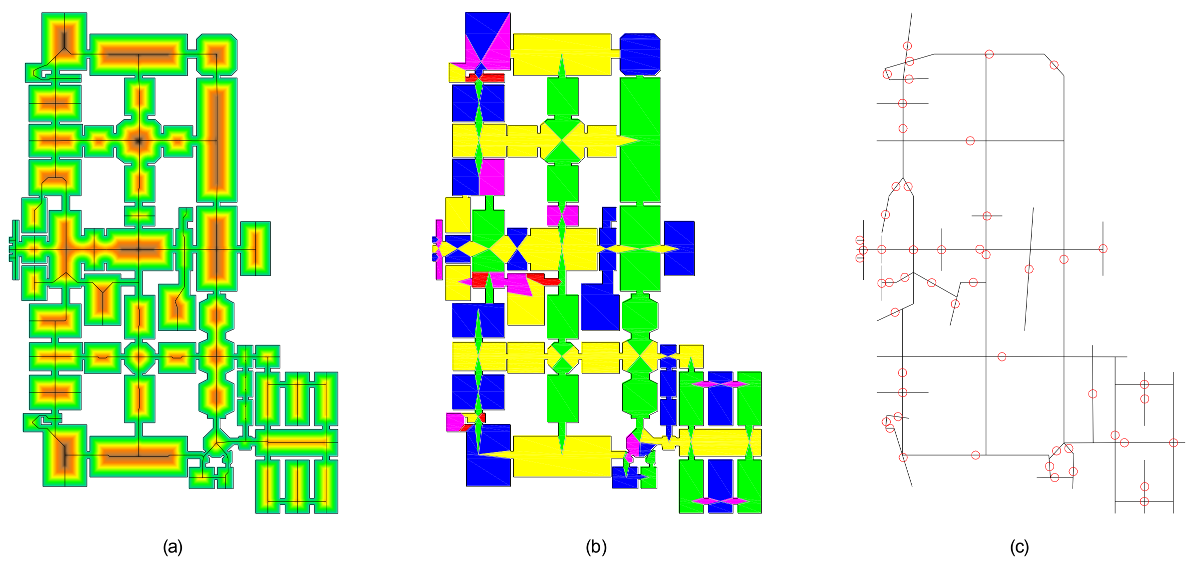

Figure 8. (a) The traced medial axes on top of the distance field map. (b) The final convex-space sets colored to differentiate. (c) The final axial lines with their midpoints.

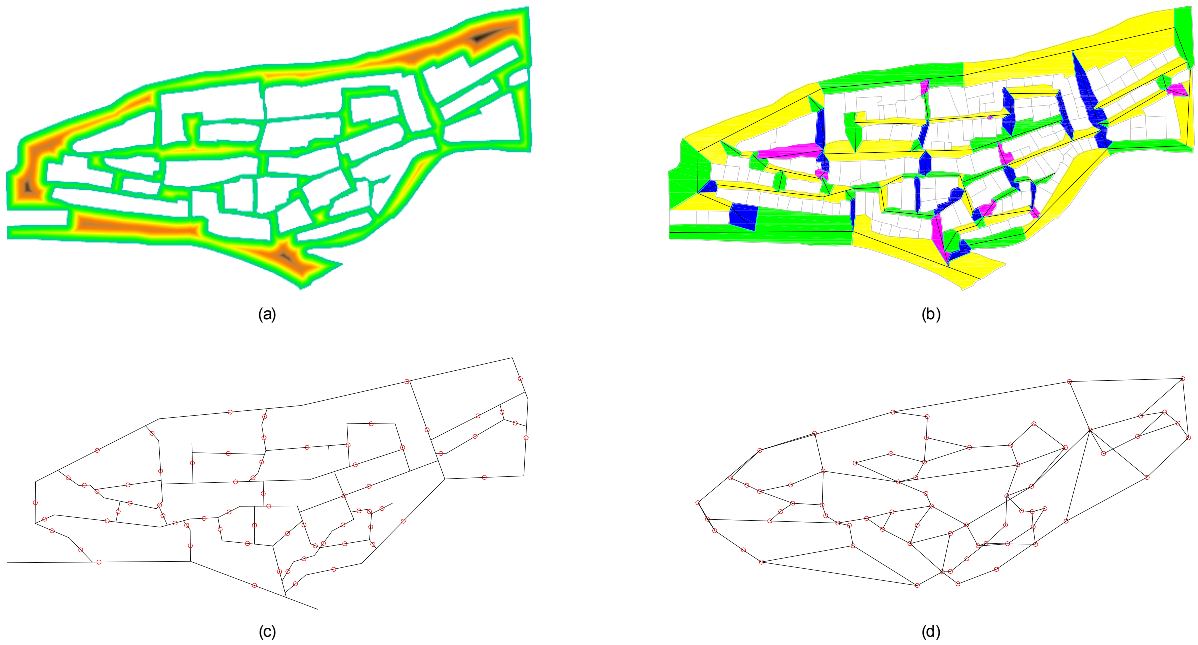

Figure 9. (a) The distance field map. (b) The final axial lines on top of the final convex-space sets colored to differentiate. (c) The final axial lines with their midpoints. (d) The final dual graph.

4.2 Thoughts on implementation

All previous results of testing the algorithm have been done manually on a computer-aided design and drafting (CADD) system. However, the algorithm is intended to be eventually an automated solution from the outset. Therefore certain aspects of the implementation have been considered thoroughly. Currently the targeted platform is the GRASS GIS for two reasons. First, because, as free and open source software, GRASS has almost all foundations required for implementing such an automated solution in place, it saves tremendous amount of efforts on rebuilding everything from scratch. Second, the author already has the experience on conducting a similar but much simpler implementation regarding space syntax calculation using GRASS (Wang & Liao 2006, 2007). The lesson learned from that particular implementation is using UNIX shell scripts alone for even such a modest computational geometry task not only simply inadequate but also too slow to be practical beyond proof-of-concept. Therefore the algorithm should be implemented as a dedicated vector command module written in the more powerful C programming language, which is also what used to build the GRASS GIS.

There are a few distinct phases in the algorithm that can be implemented individually. The first phase is the generation of medial axis. Although the r.cost raster command already provides the function to support generation of medial axis by manual tracing, the automatic generation of medial axis should be implemented as a vector command in order to output a vector map for later phases. There is abundant computational geometry related literature on algorithms for generating medial axis. Many point out the scheme that uses a set of sample points on the shape boundary and then approximates the medial axis with the Voronoi diagram of these points (Dey & Zhao 2004, p 195; Foskey et al 2003, p 276; Tam & Heidrich 2003, p 483; Chin et al 1999, pp 406-407). Because GRASS has already have a vector command called v.voronoi that generates Voronoi diagram, the source code from this command can serve as the foundation for the medial axis generating function to built upon. Even the second phase of partitioning populous convex subspaces constrained by the medial axis might be implemented by taking advantage of the existence of v.delaunay command and use its source code to build the desire functionality because the partitioning process is somewhat similar to that of generating Delaunay triangulation. The third and forth phases of creating convex map and axial map, respectively, can rely on existing GRASS library functions on measuring angle, connecting two points to form a line, checking where a line is completely inside a polygon, and merging polygons to accomplish the tasks. Finally the entire implementation can either work as a modified v.generalize command with the addition of necessary options and parameters or a new dedicated spatial network analysis command module in GRASS.

5. Conclusion

Many methods and options are available in the GRASS v.generalize vector command (GRASS Development Team 2009), but because v.generalize is not tailored to urban open space none of them take advantage of the usually associated street layout information to make generalization more effective or efficient. This paper describes a new algorithm that is specifically adapted for urban open spaces as well as spaces inside buildings and thus makes great use of the boundary information of these spaces. The boundary information is first used to generate the medial axis and then, with the assistance of medial axis, partition the whole space into convex subspaces. Then the principle of continuity is used to guide the generalization of the medial axis. By downplaying the absolutely longest axial lines but emphasizing the least angular change, the proposed algorithm not only effectively creates the axial map consisting of a slim set of generalized medial axes that still effectively resembles the original medial axis, but also efficiently creates the complementary map of convex-space sets for further analysis. Once the algorithm is implemented as an automatic solution in an open source GIS, it would be a great tool for the analysis of spatial configures of all kinds.

References

Batty, M & Rana S 2004, 'The automatic definition and generation of axial lines and axial maps', Environment and Planning B: Planning and Design, vol. 31, pp. 615-640.

Blum, H 1967, 'A transformation for extracting new descriptors of form', in Models for the Perception of Speech and Visual Form, ed W Whaten-Dunn, MIT Press, Cambridge, MA, pp. 362–380.

'Cartographic Generalization' 2010, Wikipedia, viewed 31 July 2010, <http://en.wikipedia.org/w/index.php?title=Cartographic_generalization&oldid=350481358>.

'Centrality' 2010, Wikipedia, viewed 31 July 2010, <http://en.wikipedia.org/w/index.php?title=Centrality&oldid=369388501>.

Chin, F, Snoeyink, J & Wang CA 1999, 'Finding the medial axis of a simple polygon in linear time', Discrete and Computational Geometry, vol. 21, no. 3, pp. 405-420.

Dalton, N 2001, 'Fractional configurational analysis and a solution to the Manhattan problem', in

Proceedings of the 3rd International Space Syntax Symposium, Georgia Institute of Technology, Atlanta, GA, pp. 26.1-26.13, <http://undertow.arch.gatech.edu/homepages/3sss/>.

Dalton, N, Peoponis, J & Dalton R 2003, ‘To tame a tiger one has to know its nature: extending weighted angular integration analysis to the description of GIS road-centerline data for large scale urban analysis’, in Proceedings of the 4th International Space Syntax Symposium, University College London, London, pp. 65.1-65.10, <http://www.spacesyntax.org/SSS4.htm>.

Dalton, RC 2003, ‘The secret is to follow your nose: route path selection and angularity’, Environment and Behavior, vol. 35, no. 1, pp. 107-131.

Desyllas, J & Duxbury E 2001, ‘Axial maps and visibility analysis: a comparison of their methodology and use in models of urban pedestrian movement’, in Proceedings of the 3rd International Space Syntax Symposium, Georgia Institute of Technology, Atlanta, GA, pp. 27.1-27.13, <http://undertow.arch.gatech.edu/homepages/3sss/>.

Dey, T & Zhao W 2004, ‘Approximate medial axis as a Voronoi subcomplex’, Computer-Aided Design: solid modeling and applications, vol. 36, no. 2, pp. 195-220.

Foskey, M, Lin, MC & Manocha D 2003, ‘Efficient computation of a simplified medial axis’, Journal of Computing and Information Science in Engineering. vol. 3, no. 4, pp. 274-284.

Freeman, LC 1977, ‘A set of measures of centrality based on betweenness’,. Sociometry, vol. 40, pp. 35-41.

Freeman, LC 1979, ‘Centrality in social networks: conceptual clarification’, Social Networks, vol. 1, no. 3, pp. 215-239.

GRASS Development Team 2009, GRASS 6.4 Users Manual, viewed 31 July 2010, <http://grass.osgeo.org/grass64/manuals/html64_user/>.

Haining, R 2004, Spatial data analysis: theory and practice, Cambridge University Press, Cambridge.

Hillier, B 1996, Space is the machine: a configurational theory of architecture, Cambridge University Press, Cambridge.

Hillier, B & Hanson J 1984, The social logic of space, Cambridge University Press, Cambridge.

Hillier, B & Iida S 2005, ‘Network and psychological effects in urban movement’, in Proceedings of Spatial Information Theory: international conference, COSIT 2005, Ellicottsville, NY, eds A Cohn & D Mark, Springer Berlin, Heidelberg, pp. 475-490.

Hillier, B & Vaughan L 2007, ‘The city as one thing’, Progress in Planning, vol. 67, no. 3, pp. 205-230.

Jacobs, AB 1995, Great Streets, MIT Press: Cambridge, MA.

Jiang, B & Claramunt C 2002, ‘Integration of space syntax into GIS: new perspectives for urban morphology’, Transactions in GIS, vol. 6, pp. 151-162.

Jiang, B & Claramunt C 2004a, ‘A structural approach to the model generalization of an urban street network’, GeoInformatica, vol. 8, pp. 157-171.

Jiang, B & Claramunt C 2004b, ‘Topological analysis of urban street networks’, Environment and Planning B: planning and design, vol. 31, pp. 151-162.

Jiang, B & Liu X 2010, ‘Automatic generation of the axial lines of urban environments to capture what we perceive’, International Journal of Geographical Information Science, vol. 24, no. 4, pp. 545–558.

‘Leonhard Euler’ 2010, Wikipedia, viewed 31 July 2010, <http://en.wikipedia.org/w/index.php?title=Leonhard_Euler&oldid=374465228>.

Leymarie, FF & Kimia BB 2008, 'From the infinitely large to the infinitely small: applications of medial symmetry representations of shape', in Medial Representations: mathematics, algorithms and applications, eds K Siddiqi & SM Pizer, Springer Netherlands, Houten, pp. 327-351.

McGarigal, K, Cushman, SA, Neel, MC & Ene E 2002, FRAGSTATS: spatial pattern analysis program for categorical maps, University of Massachusetts, Amherst, <http://www.umass.edu/landeco/research/fragstats/fragstats.html>.

McMaster, RB & Shea KS 1992, Generalization in Digital Cartography, Association of American Geographers, Washington, DC.

'Medial Axis' 2010, Wikipedia, viewed 31 July 2010, <http://en.wikipedia.org/w/index.php?title=Medial_axis&oldid=369586851>.

Nieminen, J 1974, 'On centrality in a graph', Scandinavian Journal of Psychology, vol. 15, pp. 322-336.

Obaid, TA 2007, State of World Population 2007: unleashing the potential of urban growth, United Nations Population Fund, New York, viewed 15 July 2010, <http://www.unfpa.org/swp/swpmain.htm>.

Penn, A, Conroy, R, Dalton, N, Dekker, L, Mottram C & Turner A 1997, 'Intelligent architecture: new tools for three dimensional analysis of space and built form', in Proceedings of the 1st International Space Syntax Symposium, University College London, London, pp. 30.1-30.19, <http://www.spacesyntax.org/symposia/SSS1/sss1_proceedings.htm>.

Peponis, J, Wineman, J, Rashid, M, Kim SH & Bafna S 1997, 'On the description of shape and spatial configuration inside buildings: convex partitions and their local properties', Environment and Planning B: planning and design, vol. 24, pp. 761-781.

Peponis, J, Wineman, J, Bafna, S, Rashid M & Kim SH 1998, 'On the generation of linear representations of spatial configuration', Environment and Planning B: planning and design, vol. 25, pp. 559-576.

Porta, S, Crucitti P & Latora V 2006, 'The network analysis of urban streets: a primal approach', Environment and Planning B: planning and design, vol. 33, pp. 705-725.

'Scale-free Network' 2010, Wikipedia, viewed 31 July 2010, <http://en.wikipedia.org/w/index.php?title=Scale-free_network&oldid=367998736>.

'Seven Bridges of Königsberg' 2010, Wikipedia, viewed 31 July 2010, <http://en.wikipedia.org/w/index.php?title=Seven_Bridges_of_K%C3%B6nigsberg&oldid=371182036>.

'Small-world Network' 2010, Wikipedia, viewed 31 July 2010, <http://en.wikipedia.org/w/index.php?title=Small-world_network&oldid=376151063>.

'Space Syntax' 2010, Wikipedia, viewed 31 July 2010, <http://en.wikipedia.org/w/index.php?title=Space_syntax&oldid=357784648>.

Space Syntax Laboratory 2008, Introduction, viewed 31 July 2010, <http://www.spacesyntax.org/introduction/index.asp>.

Strom, S, Nathan K & Woland J 2009, Site Engineering for Landscape Architects, 5th edn, John Wiley & Sons, Hoboken, NJ.

Tam, R & Heidrich W 2003, 'Shape Simplification Based on the Medial Axis Transform', in Proceedings of the 14th IEEE Visualization Conference (VIS'03), pp. 481-488.

Thomson, RC 2003, 'Bending the axial line: smoothly continuous road centre-line segments as a basis for road network analysis', in Proceedings of the 4th International Space Syntax Symposium, University

College London, London, pp. 50.1-50.10, <http://www.spacesyntax.org/SSS4.htm>.
Trancik, R 1986, Finding Lost Space: theories of urban design, John Wiley & Sons, Hoboken, NJ.
Turner, A, Penn A & Hillier B 2005, 'An algorithmic definition of the axial map', Environment and Planning B: planning and design, vol. 32, pp. 425-444.
Wang, W & Liao H 2006, 'Implementing the Space Syntax Techniques: a GRASS application for the analysis of spatial configurations', in Proceedings of the FOSS4G2006 - Free and Open Source Software for Geoinformatics conference, Open Source Geospatial Foundation, Lausanne, Switzerland, <http://2006.foss4g.org/contributionDisplay4ab3.html?contribId=97&sessionId=49&confId=1>.
Wang, W & Liao H 2007, 'Implementing Space Syntax in an Open Source GIS: GRASS GIS approach', in Proceedings of the 6th International Space Syntax Symposium, Istanbul Technical University, Istanbul, pp. 100.1-100.14, <http://www.spacesyntaxistanbul.itu.edu.tr/>.