Cartographic Projection Procedures
Release 4
Interim Report

Gerald I. Evenden

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Contents

Introduction

This is an interim document introducing changes and additions to release 4 of the cartographic projection program **proj** originally described in *Cartographic Projection Procedures for the* UNIX *Environment—A User's Manual* (U.S. Geological Survey Open-File Report 90–284). Because this report adds to, and does not replace, 90-284, new users of this system should obtain copies of the original report for full documentation of the program. Users of release 3 **proj** should pay careful attention to details of this new release which may affect current scripts and usage.

The principle reason for release 4 of **proj** is to increase the system's portability and usability. Two prime factors are considered in attempting to achieve this goal:

1. to make the C language source code compatible and compliant with ANSI language standards and POSIX procedural standards and

2. improve the modularity and encapsulization of the internals.

Although the earlier version, coded in K&R style C, was generally successful in installation, occasional problems occurred that were due to site system peculiarities. Hopefully, most of these have been eliminated.

Although the program **proj** is a reasonably flexible filter tool, it is limited in its application to tasks that lend themselves to this mode of data processing. To help software developers that need cartographic procedures embedded in their programs, the cartographic procedures used in **proj** have been more carefully encapsulated and thus make their inclusion in other application software a relatively easy task. Individual projection procedures can now also have multiple states of initialization so that processes such as datum transformations can be carried out within the same program.

Acknowledgements

The author expresses his gratitude to the large number of individuals who have contributed to the improvements and refinements of this software through questions, suggestions and an occasional complaint. In particular, Jerry L. Bohannon has made several suggestions that have been incorporated in the current release and has supplied valuable source material. User feedback is a prime requirement in any attempt to develop quality software.

In addition, special thanks to John P. Snyder for resolving technical problems and supplying additional source material.

Release 3–4 Compatibility

Despite losing some upward compatibility, a few executional changes of release 4 of **proj** were necessary in order for options to maintain a reasonable relationship with the revised internals of the system. Two **proj** control parameters found in earlier releases are deleted: the `-c` for naming a source of ancillary control data and `+inv` for specifying the inverse mode. The action of the `-c` option is replaced by the more versatile initialization files and the `+init` parameter. Specifying inverse projections is now done with the `-I` parameter. Inverse projections with **invproj** name remains in effect.

Of lesser importance, the use of `list` as an argument to the `+ellps` and `+proj` to obtain a listing of the available ellipsoid constants and projections has been dropped from release 4. Run-line options `-le` and `-lp` now perform these respective functions.

New hyphen options.

To obtain a list of **proj** projections, the `-l` or `-lp` option will display a list of all projections supported in the current installation (replaces the former `+proj=list` option). An list with expanded explanation of each projection and associated parameters is obtained by using `-lP`. Examples of these option:

```
proj -l
qua_aut : Quartic Authalic
aea : Albers Equal Area
aeqd : Azimuthal Equidistant
airy : Airy
aitoff : Aitoff
alsk : Mod. Stererographics of Alaska
...
```

and

```
proj -lP
qua_aut : Quartic Authalic
    PCyl., Sph.
aea : Albers Equal Area
    Conic Sph&Ell
    lat_1= lat_2=
aeqd : Azimuthal Equidistant
    Azi, Sph&Ell
    lat_0= guam
...
```

In general, the first supplementary line describes projection class (pseudocylindrical, conic, ...), spherical or elliptical, ..., and additional lines list options unique to each projection.

For a short reminder of options associated with a single projection, the option `-l=`*id* can be used where *id* is the accronym of the projection in question. For example:

```
proj -l=lcc
   lcc : Lambert Conformal Conic
     Conic, Sph&Ell
     lat_1= and lat_2= or lat_0=
```

Because **proj** may be using the initialization and default files (see Runtime Initialization Files) the user may not be aware of the actual parameters being used by **proj**. In addition, parameter misspelling or faulty usage can go unnoticed because **proj** does not flag nor notify the user of parameters it does not know about. The `-v` option is used to help verify selection and usage of projection parameters (+ parameters) by displaying what values were actually used by the program. In addition, parameters that were entered but not used are also noted and listed. For example, the user performs the following **proj** execution:

```
# proj +proj=poly +lat_0=40 +lon0=-66 -v
```

with the following results printed by the `-v` printed at the beginning of the output:

```
# +proj=poly +lat_0=40 +ellps=clrk66
# following specified but NOT used
# +lon0=-66
```

The +lon0 parameter was not used and the user probably intended to use +lon_0. Although the user might have sensed an error by examining the output and seeing questionable values, other errors can be more subtle and difficult to detect. Also note, the user is informed of the ellipsoid that was selected by the proj_def.dat file.

The -E option is added as a convenience by causing the input coordinates to be copied to the output stream prior the printing the projected results. Thus the both forward and inverse values are placed side by side on the output shown in this example output:

```
# sample points
65W 43d15N      -405817.61      4802414.53
-55 37.33       442931.70       4144652.95
```

created by the following script:

```
proj +proj=poly +lon_0=-60 -E <<EOF
# sample points
65W 43d15N
-55 37.33
EOF
```

When developing a new map or region for a plane coordinate system it is desirable to adjust the projection parameters to minimize the projection distortion over the area. Although analytic methods may be used to determine these factors it is often as easy to "cut and try" if a means to quickly check these values is available. Scale and information on other factors is important when using information in cartesian space. To provide information about the performance of a projection at a point the -V option provides an anotated lists of scale factors and other factors at each location entered. Executing the following lines to determine characteristics The following execution of **proj** shows the use of this switch for a point in the Massachussetts Mainland SPCS zone:

```
proj +init=nad27:2001 +units=us-ft -V
-70d36'30.872 41d38'54.192 A residence
```

which will produce the following output from **proj**:

```
Lambert Conformal Conic
    Conic, Sph&Ell
    lat_1= and lat_2= or lat_0
+init=/usr/local/lib/proj/nad27:2001 +units=us-ft +proj=lcc +a=6378206.4
+es=.006768657997291094 +lon_0=-71d30 +lat_1=42d41 +lat_2=41d43 +lat_0=41
+x_0=182880.3657607315 +y_0=0 +no_defs
Final Earth figure: ellipsoid
  Major axis (a): 6378206.400
  1/flattening: 294.978698
  squared eccentricity: 0.006768657997
 A residence
Longitude: 70d36'30.872"W [ -70.608575556 ]
Latitude:  41d38'54.192"N [ 41.648386667 ]
Easting (x):   843640.74
Northing (y):  237542.45
Meridian scale (h) : 1.00001069   ( 0.001069 % error )
Parallel scale (k) : 1.00001069   ( 0.001069 % error )
Areal scale (s):     1.00002138   ( 0.002138 % error )
Angular distortion (w): 0.000
Meridian/Parallel angle: 90.00000
Convergence : -0d35'55.663" [ -0.59879536 ]
Max-min (Tissot axis a-b) scale error: 1.00001 1.00001
```

The "Final Earth figure" is shown to inform the user as to the effect of either selecting one of the `+R` options or the fact that the projection is only for a spherical figure. Discrepancies of the h and k values, which should be exactly 1., are due to the limitations of determining derivatives by numeric rather than analytic methods. To maintain a comple complete information log, the `-v` option is implicit with `-V`.

Additional points result in similar output and the user can also override the forward-inverse mode of **proj** by making the the first character of a data line either an upper of lower case `f` for forward or `i` for inverse. Any information after the coordinates, such as the notation "`A residence` " in the previous, example, are printed out before the analytic information.

The meridian, h, and parallel, k, scale factors are the respective scales along the meridian and parallel through the point and the areal scale factor, s, is the area scale at the point. For conformal projections, $h = k$ for all points and for equal-area projections s will be constant for all points.

If two lines pass though the point the angle between these lines in geographic space may be as much as twice the angular distortion, 2ω, different in cartesian space. Angular distortion is a common metric to quantify distortion of non-conformal maps by contouring either ω or 2ω. Angular distortion is always 0 for conformal projections. Meridian-parallel angle, θ', is the angle between the meridian and parallel in cartesian space and is always 90° for conformal projections.

The convergence angle, γ, is the angle measured from the positive y cartesian axis of the projection to true North—the meridian through the point. Most large scale maps, such as the USGS quadrangle series, will have a margin figure with UTM grid and magnetic declination measured at the center of the sheet. "Grid declination" is the convergence angle.

Scale factors a and b are the maximum and minimum scale error and define the major and minor axis of the Tissot Indicatrix. The Tissot Indicatix, used as a visual indication of map distortion, is based upon the concept of drawing a small circle in geographic space and then portray a magnified image of this circle after it has been projected. For conformal maps, the Indicatrix will remain a circle but have a different size depending upon its location on the map. Equal-area maps will have circular Indicatrixs at a point or along a line, but they will be elliptical in shape elsewhere with only the area withing the ellipse remaining constant.

The `-S` option provides a summary of the above information as a field of values, enclosed by `<>`, appended to the output record. Values incude meridinal and parallel scale scale factors (h and k), area scale factor (s), angular distortion (ω) in decimal degrees, and the major and minor axis of the Tissot Indicatrix (a, b).

Two projections of the conterminous U.S., Alber's and Lambert Conformal Conic, demonstrate characteristics of the `-S` option output as related to respective equal area and conformal projections. Starting with the script for Albers Equal Area:

```
proj -S +proj=aea +lon_0=90W -v <<EOF
-73 37
-110 44
EOF
proj -S +proj=lcc +lon_0=90W -v <<EOF
-73 37
-110 44
EOF
```

the following output was obtained:

```
# +proj=aea +lon_0=90W +ellps=clrk66 +lat_1=29.5 +lat_2=45.5
 1490786.23 4043351.48  <1.00965 0.990439 1 0.550448 0.990439 1.00965>
-1586582.09 4860774.53  <1.00364 0.996375 1 0.208089 0.996375 1.00364>
# +proj=lcc +lon_0=90W +ellps=clrk66 +lat_1=33 +lat_2=45
```

```
 1497189.34 4543009.70  <0.995191 0.995191 0.990405 0 0.995191 0.995191>
-1588520.83 5351853.03  <0.998284 0.998284 0.996571 0 0.998284 0.998284>
```

Note how the area scale factor (third term) remained unchanged for both points as would be expected for an equal area projection but angular distortion and both scale factors vary. In the case of the Lambert projection, the scale factors will vary between points but for a particular point they are always equal in both directions and the angular distortion is always zero. This example shows the defining properties of the equal area and conformal projections.

Radius Parameters

In previous releases of **proj** the radius of a spherical earth figure was specified by the major axis parameter `+a` and either an explicit or implicit specification of `+es=0` for those projections with elliptical form. In release 4 the radius of a spherical Earth may be entered with the `+R=`*radius* and thus bypassing an unnecessarily complex method. Use of `+R=` takes precedence over any elliptical parameter specifications so that their possible appearance in the control parameter list is ignored.

Because of the need to specify an Earth radius that has a relationship with an ellipsoid, a set parameters are introduced to compute this radius when an ellipsoid is also selected. The projection computations will be treated as a sphere when one of these parameters is selected:

`+R_A`	Radius of a sphere with equivalent surface area of specified ellipse.
`+R_V`	Radius of a sphere with equivalent volume of specified ellipse.
`+R_a`	Arithmetic mean of the major and minor axis, $R_a = (a + b)/2$.
`+R_g`	Geometric mean of the major and minor axis, $R_g = (ab)^{1/2}$.
`+R_h`	Harmonic mean of the major and minor axis, $R_h = 2ab/(a + b)$.
`+R_lat_a=`ϕ	Arithmetic mean of the principle radii at latitude ϕ.
`+R_lat_g=`ϕ	Geometric mean of the principle radii at latitude ϕ.

As an example, the Albers Equal Area projection is to be computed in the spherical form for an Earth with a radius such that the sphere has the same surface area as the Clarke 1866 ellipsoid:

```
# proj +proj=aea +ellps=clrk66 +R_A ...
```

For projections that only perform computations for a sphere, this method is preferable to simply specifying an ellipsoid and thus having the projection use the major axis as the radius. The order of the radius and ellipsoid parameters is not important.

Cartesian Units

Basic operation of **proj** assumes that projected cartesian units are the same units as the lengths associated with the projection parameter units (*i.e.* `+a`, `+b`, `+x_0`, ...) which are normally in meters. For some usage, such as for SPCS computations, it is useful to provide forward-inverse conversion between geographic coordinates and other, non-meter, systems such as feet. Usage of the parameter `+units=`*id* allows specification of several alternative of length measure. For example, if U.S. feet are desired then the parameter `+units=us-ft` is used as a parameter and the cartesian coordinates output in the forward mode and input in the inverse mode are in feet. Usage of this parameter **does not** affect the units of the + projection parameters—they must be in meters when using `+units`. The current list of units supported can be obtained by using **proj**'s run-line option `-lu`:

```
   km 1000.              Kilometer
    m 1.                 Meter
   dm 1/10               Decimeter
   cm 1/100              Centimeter
   mm 1/1000             Millimeter
  kmi 1852.0             International Nautical Mile
   in 0.0254             International Inch
   ft 0.3048             International Foot
   yd 0.9144             International Yard
   mi 1609.344           International Statute Mile
 fath 1.8288             International Fathom
   ch 20.1168            International Chain
 link 0.201168           International Link
us-in 1./39.37           U.S. Surveyor's Inch
us-ft 0.304800609601219  U.S. Surveyor's Foot
us-yd 0.914401828803658  U.S. Surveyor's Yard
us-ch 20.11684023368047  U.S. Surveyor's Chain
us-mi 1609.347218694437  U.S. Surveyor's Statute Mile
```

The numeric value listed for reference purposes and is the value used to convert the users cartesian coordinates to and from meters used for internal computations:

$$(x, y)_{meters} \quad \leftrightarrow \quad conv \times (x, y)_{usersunits}$$

There is considerable variety of units of length measure and to include all units used in just the last 200 years would only create confusion for the user. Other situations such as the fact that the brass bar that established the standard for the British yard shrank in length between 1853 and 1958 by about 5.5μ (Bomford, 1971) add to this confusion. Although such a small error seems trivial, it does cause problems with high precision calculations associated with plane coordinate systems. These factors along with the difficulty in resolving differences in conversion factors for less common units the `+units` list is restricted to recent and well defined conversions.

In order to allow other conversions to be imbedded within the cartographic control parameters and thus be part of initialization and default control files the `+to_meter=`*frac* may be used. The value of *frac* is a numeric value with properties identical to those of the conversion number listed with the `-lu` **proj** option. As shown in the `-lu` listing, the value may be expressed as a rational fraction with the numerator and denominator separated with a `/`.

Initialization Parameter

Common usage of certain projections or projection features may be facilitated by the projection parameters being predefined in initialization files. They are accessed by the parameter `+init=`*file:key* where *file* is the name of the file containing the control information and *key* identifies the particular set of parameters in the file to be included as projection parameters. Conversion of SPCS data (see ref) is case for U.S. users where details of plane coordinate conversion are located in initialization files. For example, to convert 1927 North American Datum Massachussetts Mainland coordinates to geographic coordinates:

```
$ proj -I +init=nad27:2001 <in_data >out_data
```

The file `nad27` contains projection parameters for NAD27 conversions and the key `2001` refers to the particular entry needed. Program **proj** will complain if either the file cannot be found or there is no keyword or keyword data in the file.

Initialization files may be established by site personel responsible for **proj** administration or created by the individual user. Administrator files are located in a directory specified by the user's environment parameter `PROJ_LIB` and it is the responsibility of the administrator to distribute documentation and instructions of file contents and correct usage. Unless the administrator gives permission to the user to install his files in the system area, the user will have to refer to his initialization file with an absolute path:

```
$ proj +init=/home/me/lib/my_defs:proj5 ...
```

For UNIX users, the ~ prefix to the file name will prepend the contents of the `HOME` environment parameter. The user should refer to the next section on creating initialization files.

Runtime Initialization and Default Files

Program **proj** is designed with runtime facilities to configure application definitions and default parameters to the needs of the local environment. This is achieved though the usage of two types of ASCII text control files, initialization and default, that are coded in a very simple control syntax that is identical for both types of files—only the keyword usage differs.

Structure of the control files consists of identifiers in the form <*keyword*> followed by a sequence of projection parameters. When processing the control file **proj** scans for the specified keyword and when found, adds the parameters following the keyword to the internal control list. Processing of parameters continues, ignoring the occurrence of other keywords, until the `<>` character pair is encountered. When `<>` is found after the desired keyword, processing of the control file is stopped, thus a second occurrence of the keyword in the file is ignored. As with run-line projection parameters, keywords and parameters are words that are groups of characters that are separated by blanks, tabs or newlines. When a word begins with a `#` character all input is ignored until the next newline character; thus comments may be added to describe the data.

The following is a simple example of a initialization control file:

```
# a sample (comment line)
<myid1> proj=tmerc Ra <> # spherical
                 # transverse mercator
<pj-sph> Ra      # spherical form of
                 # the following
<pj-ell> proj=poly lon_0=90 ellps=airy <>
```

When the keyword `myid1` is used the projection is set and the sphere of area equivalent to ellipse is selected but the ellipse and other parameters needed and must be input by other means. The next two identifiers give sufficient detail to allow *proj* to perform the projection. The `pj-sph` is an example where the second keyword is ignored and its parameters are included as part of the first keyword specifications.

The first of the two types of control files are used by **proj** is the initialization file explicitly referenced by the user with the `+init` control parameter. Its purpose is to provide a convenient method to define commonly used and complex sets of control parameters for map or grid coordinate system. For example, the standard zones for the SPCS systems are contained in two distributed initialization files `nad27` and `nad83`. Typically, the projection selection parameter, `proj`, is contained in these files and there are sufficient parameters to fully qualify all options associated with the projection.

Unless the `+no_defs` projections parameter has been given, the second control file (defaults file) is processed after all other projection parameters have been input and after the projection name has been established. It is scanned for two keywords: `general` and a keyword that is the name of the selected projection. Parameters associated with `general` are default values associated with all projections and typically defines a default ellipsoid. Projection parameters are those normally associated with that projection in a particular geographic area of usage. A typical example (from the **proj** distribution) would be:

```
<general> # for all projections
ellps=clrk66 # ellipsoid compatible
             # with older U.S. maps
<>
<aea> # Conterminous U.S. map
lat_1=29.5
lat_2=45.5
<>
<lcc> # Conterminous U.S. map
lat_1=33
lat_2=45
<>
   ...
```

The name of this file is `proj_def.dat` and is located in the directory established by the installer or pointed to bye the environment parameter `PROJ_LIB` (see next section). For non-U.S. installations, it should be edited by the installer to reflect local cartographic customs and usage. Program **proj** continues processing if the file cannot be found or opened and in certain cases projection initialization will fail.

Paths of control files

The location of the initialization control file is controlled by how the user names the `+init` file, how program **proj** was installed and the optional presence of the environment parameter `PROJ_LIB`. If the file name begins with a `/` the file is assumed to have an fully pathed name from the system root directory. If the name starts with `./` or `../` is not defined, the file path is treated as relative to the current working directory. When `~/` prefixes the file name, the users home directory, as defined by the environment parameter `HOME`, is used as the root of the file name.

When simple initialization file names are used (those names without aforementioned prefixes) and in the case of the automatic default file, the location of the files is controlled by **proj** installation or the user's environment parameter `PROJ_LIB`. In case of the environment parameter, the user is overriding the installation defaults and establishing his or her own initialization and default definition file path. To set the environment path do either

```
setenv PROJ_LIB /usr/local/lib/proj
```

when using **csh(1)** or

```
PROJ_LIB=/usr/local/lib/proj
export PROJ_LIB
```

when using **sh(1)** or **ksh(1)**. If the user always wants these settings, then they can be included in the **.login** or **.profile** files.

Caveats

The initialization and default files provide a useful tool to configure **proj** to a wide variety of conditions that best fit local needs and thus ease the usage of **proj** in the performance of routine tasks by less knowledgeable and infrequent users. But care should be exercised in their usage. Certain options may be included in the automatic file that may cause hidden and unintended operations. For example, inclusion of parameters such as `R_A` or `R_V` in the automatic files may cause unintended spherical computations when it was thought that an elliptical projection was explicitly specified.

Datum Conversions

The use of satellites and other technologic improvements in first order surveying have allowed geodesists to refine the knowledge of the shape of the Earth. Along with these refinements came the inevitable process of standardizing the definition of the approximating ellipsoid and establishing an international reference datum. Prior to this, the ellipsoids and datums were established by long line precision surveying and astronomical observation. The processing of the measurements of these surveys let to establishment of ellipsoids which were best fits to local conditions and not the entire Earth and datums which were arbitrary to the surveyor's network. But because this surveying relied upon the use of the spirit level for alignment of instruments with the horizontal plane (the geoid) they were susceptible to perturbations of the gravity field and thus only useful for local purposes.

Until recently, the reference system for North America has been the North American Datum of 1927 (NAD27) which used Clarke's 1866 ellipsoid and had its origin at Meade's Ranch in Kansas. But because of technical geodetic surveying problems with NAD27 and an interest in standardizing the reference system on an international basis, the North American Datum of 1983 reference system NAD83 has been chosen to replace NAD27. This system is based upon the Geodetic Reference System of 1980 (GRS80) which is geocentric (origin is the center of the Earth's mass) and uses an ellipsoid approximating the entire Earth.

There are several methods for conversion of geographic data between datums but the most convenient and perhaps common are the Molodensky formula and the NADCON (Dewhurst, 1990) used for North American Datum conversions. The Molodensky method is often used for international conversions but is considered to only have a conversion accuracy of 5–10m in United States regions. The NADCON method uses of a grid of longitude–latitude corrections from which a correction value can be interpolated for any non-nodal point. The correction grid is determined by minimum curvature gridding of corrections for control points whose location had been accurately determined by both NAD27 and NAD83 surveying methods. Error in conversion with NADCON is generally considered to be less than a meter (0.15m for most of the `conus` region) but may suffer in regions of poor control. Table 1 is a summary of the NADCON grid regions.

Table 1: NADCON correction regions.

Region	**nad2nad** `-r` *region*	Extent East	West	South	North
Conterminous U.S.	`conus`	131° W	63° W	20° N	50° N
Alaska	`alaska`	166° E	128° W	46° N	77° N
Hawaii	`hawaii`	161° W	154° W	18° N	23° N
Puerto Rico and Virgin Islands	`prvi`	68° W	64° W	17° N	19° N
St. George Is., AK	stgeorge	171° W	169° W	56° N	57° N
St. Lawrence Is., AK	srlrnc	172° W	68° W	62° N	64° N
St. Paul Is., AK	stpaul	171° W	169° W	57° N	58° N
High Precision GPS Network					
Florida	`FL`	88° W	80° W	24° N	32° N
Maryland	`MD`	80° W	74° W	37° N	41° N
Tennessee	`TN`	91° W	81° W	34° N	38° N
Washington–Oregon	`WO`	125° W	116° W	41° N	50° N
Wisconsin	`WI`	94° W	88° W	42° N	48° N

Recent releases (circa July, 1993) of NADCON tables also include tables for conversion between the High Precision GPS Networks (HPGN) and NAD83. Little information about the HPGN was distributed with the tables so usage is available but not defined at the moment. These tables are for state regions.

Program nad2nad.

For conversion of data between NAD27 and NAD83 datums the software distribution now includes the program **nad2nad**. It performs in a manner similar to program **proj** and has several of the same runline options so users familiar with **proj** should have little trouble with learning **nad2nad**. Besides performing datum conversions it will perform SPCS and UTM conversions for both input and output thus allowing both geographic as well as grid data to be processed.

The internal functioning of **nad2nad** is a three step process:

1. process input data and, if selected, convert data from grid system coordinates to geographic coordinates,
2. if NADCON region selected, convert geographic data between datums, and
3. process output data and, if selected, convert to grid system coordinates.

Control of the input and output steps are by means of the respective `-i` and `-o` runline options which have an identical list of arguments:

`27` — data is in NAD27 datum. This is the default state.

`83` — data is in NAD83 datum.

`utm=`*zone* — data in UTM coordinates for identified zone (numeric value between 1 and 60).

`spcs=`*zone* — data in SPCS coordinates for identified State zone (see Table 2).

`bin` — data in binary form.

`rev` — reverse normal longitude-latitude or x-y order of data.

`feet` — data is in U.S. Surveyor's feet, otherwise in meters. Must be used in conjunction with `spcs` option.

`hpgn=`*zone* — data is in HPGN datum for zone listed in Table 1.

These options represent the state of the data at respective input and output of steps 1 and 3 and thus determine the necessary actions to be taken to convert the information to intermediate geographic coordinates required for datum shift. More than one option can be used and in this case they may be in a comma separated list or separate `-i` or `-b` options as shown by the following:

```
# nad2nad -i 83 -i spcs=1001 -i feet ...
# nad2nad -i 83,spcs=1001,feet ...
```

Option order is not important.

Step 2 of **nad2nad** is controlled by the `-r` <*region*> option which determines which NAD27–NAD83 zone listed in Table 1 is to be used. When this option is specified the the `-i` and `-o` must indicate different datums, thus

```
# nad2nad -i 27 -o 83 -r conus ...
```

is correct usage, while

```
# nad2nad -i 27 -o 27 -r conus ...
# nad2nad -r conus ...
```

are incorrect usage. The following is an example where geographic NAD27 coordinates are to be converted to geographic NAD83 coordinates:

```
# nad2nad -i 27 -o 83 -r conus <<EOF
-71d15 44d20'15
120W 30N
87d30 52d14
EOF
```

which produces the output:

```
71d14'58.27"W   44d20'15.227"N
120d0'3.181"W   30d0'0.348"N
*       *
```

Note that the last coordinate is outside the `conus` region.

Because changing datums of grid system data is common, the **nad2nad** `utm` and `spcs` options may be used to process these systems. In this case, Massachussetts Mainland zone NAD27 coordinates in feet are converted to NAD83 values in meters by:

```
# nad2nad -i 27,spcs=2001,feet -o 83,spcs=2001 -r conus <<EOF
840000 230000
EOF
```

with the results being:

```
273193.78       820117.57
```

Similarly, the same data can be converted to UTM, zone 19 coordinates by:

```
# nad2nad -i 27,spcs=2001,feet -o 83,utm=19 -r conus <<EOF
840000 230000
EOF
```

resulting in output of:

```
364916.74       4609733.79
```

The `-r` option may be omitted so that there is no datum transformation. This allows **nad2nad** to be used for purposes such as converting SPCS grid coordinates to and from UTM grid coordinates, conversion of grid coordinates from one zone to an adjacent zone, or simply converting geographic coordinates to and from DMS and decimal degrees formats. The previous example could be a simple conversion from SPCS to UTM in the NAD27 datum as performed by:

```
# nad2nad -i 27,spcs=2001,feet -o 27,utm=19 <<EOF
840000 230000
EOF
```

with the results:

```
364869.08       4609509.76
```

To do this operation with **proj** would create considerably more system overhead due to two copies of the program executing and data piping operations.

Table 2: List of State Plane Coordinate System Zones (SPCS) and identification numbers for 1927 and 1983 North American Datums.

State Zone	'27	'83	State Zone	'27	'83	State Zone	'27	'83
Alabama East	101	101	Iowa North	1401	1401	North Carolina	3200	3200
West	102	102	South	1402	1402	North Dakota North	3301	3301
Alaska Zone 1	5001	5001	Kansas North	1501	1501	South	3302	3302
Zone 2	5002	5002	South	1502	1502	Ohio North	3401	3401
Zone 3	5003	5003	Kentucky North	1601	1601	South	3402	3402
Zone 4	5004	5004	South	1602	1602	Oklahoma North	3501	3501
Zone 5	5005	5005	Louisiana North	1701	1701	South	3502	3502
Zone 6	5006	5006	South	1702	1702	Oregon North	3601	3601
Zone 7	5007	5007	Offshore	1703	1703	South	3602	3602
Zone 8	5008	5008	Maine East	1801	1801	Pennsylvania North	3701	3701
Zone 9	5009	5009	West	1802	1802	South	3702	3702
Zone 10	5010	5010	Maryland	1900	1900	Rhode Island	3800	3800
Arizona East	201	201	Massachusetts Mainland	2001	2001	South Carolina		3900
Central	202	202	Islands	2002	2002	North	3901	
West	203	203	Michigan East	2101		South	3902	
Arkansas North	301	301	Central/m	2102		South Dakota North	4001	4001
South	302	302	West	2103		South	4002	4002
California I	401	401	North	2111	2111	Tennessee	4100	4100
II	402	402	Central/l	2112	2112	Texas North	4201	4201
III	403	403	South	2113	2113	North Central	4202	4202
IV	404	404	Minnesota North	2201	2201	Central	4203	4203
V	405	405	Central	2202	2202	South Central	4204	4204
VI	406	406	South	2203	2203	South	4205	4205
VII	407		Mississippi East	2301	2301	Utah North	4301	4301
Colorado North	501	501	West	2302	2302	Central	4302	4302
Central	502	502	Missouri East	2401	2401	South	4303	4303
South	503	503	Central	2402	2402	Vermont	4400	4400
Connecticut	600	600	West	2403	2403	Virginia North	4501	4501
Delaware	700	700	Montana		2500	South	4502	4502
Florida East	901	901	North	2501		Washington North	4601	4601
West	902	902	Central	2502		South	4602	4602
North	903	903	South	2503		West Virginia North	4701	4701
Georgia East	1001	1001	Nebraska		2600	South	4702	4702
West	1002	1002	North	2601		Wisconsin North	4801	4801
Hawaii 1	5101	5101	South	2602		Central	4802	4802
2	5102	5102	Nevada East	2701	2701	South	4803	4803
3	5103	5103	Central	2702	2702	Wyoming East	4901	4901
4	5104	5104	West	2703	2703	East Central	4902	4902
5	5105	5105	New Hampshire	2800	2800	West Central	4903	4903
Idaho East	1101	1101	New Jersey	2900	2900	West	4904	4904
Central	1102	1102	New Mexico East	3001	3001	American Samoa	5300	
West	1103	1103	Central	3002	3002	Guam Island	5400	
Illinois East	1201	1201	West	3003	3003	Puerto Rico, Virgin Is.		5200
West	1202	1202	New York East	3101	3101	1	5201	
Indiana East	1301	1301	Central	3102	3102	(St. Croix) 2	5202	
West	1302	1302	West	3103	3103			
			long island	3104	3104			

New and Revised Projections

The seven new projections that have been added to release 4 of program **proj** are listed in Table 3. Graphic examples are shown in Figures 1–4. In addition, new options have been added to the some of the existing projections as shown in Table 4.

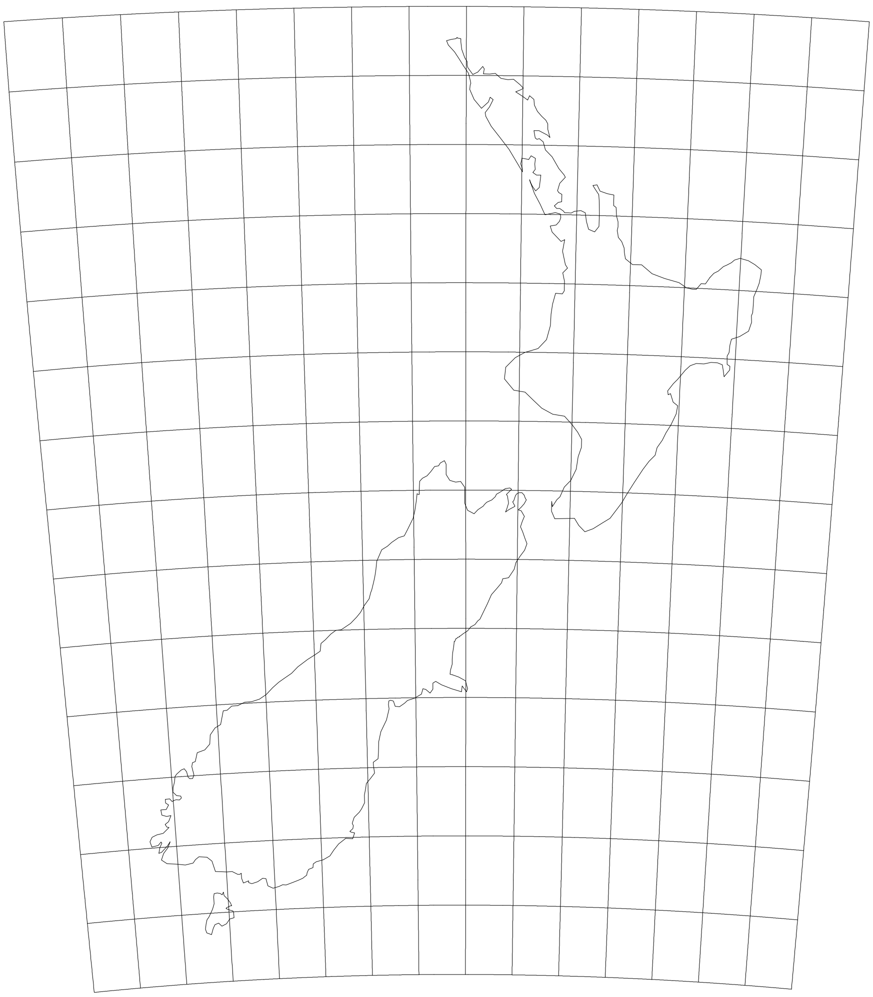


Figure 1: New Zealand Map Grid projection, with shorelines and 1°graticule.

Table 3: Projections new to release 4 of program **proj**

Projection Name (Alias)	Type*	Parameters	Comments
Two Point Equidistant (Doubly Equidistant)	S I	`+proj=tpeqd` `+lon_1=`λ_1 `+lat_1=`ϕ_1 `+lon_2=`λ_2 `+lat_2=`ϕ_2	The central points, $P(\lambda_1,\phi_1)$ and $P(\lambda_2,\phi_2)$, are on a great circle coincident with the cartesian x-axis and the cartesian origin is midway between the central points and y is positive to the left of the line from P_1 to P_2. Distance from any point to the two central points is true great circle (geodesic) distance. Scale is correct along the line through P_1–P_2. See Figure 3.
New Zealand Map Grid	C E I	`+proj=nzmg`	The central meridian (`+lon_0`) and parallel (`+lat_0`) are fixed at 173°E and 41°S respectively and the International (`+ellps=intl`) elliptical figure is fixed. False easting and northings are also fixed at (`x_0=`)2,510,000m and (`y_0=`)6,023,150m. See Figure 1.
LANDSAT	C E S I	`+proj=lsat` `+lsat=`n `+path=`p	This projection (not shown) is for use with LANDSAT satellite data and is a limited form of the more general Space Oblique Mercator projection. The LANDSAT satellite number, n, must be in the range 1–5 and the path number, p, must be in the ranges 1–251 for $n = 1, 2, 3$ or 1–233 for $n = 3, 4$.
50 United States Modified Stereographic	C E S I	`+proj=gs50`	The central meridian (`+lon_0`) and parallel (`+lat_0`) are fixed at 120°W and 45°N respectively. Selection of ellipsoid or spherical conversion is performed by conventional means, but actual values used are fixed at respective Clarke 1866 and its equivalent sphere radius, 6,370,997m. See Figure 4B.
Alaska Modified Stereographic	C E S I	`+proj=alsk`	The central meridian (`+lon_0`) and parallel (`+lat_0`) are fixed at 152°W and 64°N respectively. Control of elliptical-spherical figure is fixed an performed in an identical manner to the above 50 U.S. Modified Stereographic. See Figure 4A.
Lee Oblated Stereographic	C S I	`+proj=lee_os`	The central meridian (`+lon_0`) and parallel (`+lat_0`) are fixed at 165°W and 10°S respectively. See Figure 4D.
Miller Oblated Stereographic	C S I	`+proj=mill_os`	The central meridian (`+lon_0`) and parallel (`+lat_0`) are fixed at 20°E and 18°N respectively. See Figure 4C.
Laborde	C E I	`+proj=labrd` `+azi=`A_z `k_0=`k_0	This projection is only used for the Madagascar Grid Map (see Figure 2) where the parameters should always be specified as: `ellps=intl`, `lon_0=46d2613.95E'`, `lat_0=18d54S`, `azi=18d54`, `k_0=.9995`, `x_0=400000` and `y_0=800000`

* C–Conformal, A–Equal-Area, S–spherical, E–elliptical, I–inverse

Table 4: Projections revised in release 4 of program **proj**

Projection Name (Alias)	Type*	Parameters	Comments
Mercator (Wright)	C E I	`+proj=merc` `+lon_ts=`ϕ_{ts} or `+k_0=`k_0	Applications should be limited to equitorial regions, but it is frequently used for navigational charts with true scale (ϕ_s) specified within or near the chart's boundary. Alternatively, equitorial scale may be adjusted by specifying k_0. When neither is specified, scale is true at the Equator.
Lambert Conformal Conic (Conical Orthomorphic)	C E I	`+proj=lcc` `+lat_0=`ϕ_0 secant `+lat_1=`ϕ_1 `+lat_2=`ϕ_2 tangent `+lat_1=`ϕ_1 `+k_0=`k_0	In the secant case, ϕ_1 and ϕ_2 are the latitudes of intersection of the cone with the ellipsoid or sphere and for the tangent case, ϕ_1 is the latitude of tangency of the cone with the ellipsoid or sphere. Scale is true at the secant or tangency latitudes. The special cases where $\phi_1 = -\phi_2$ (secant mode) or $\phi_1 = 0$ (tangent mode) that configure a cylinder are not allowed. Use Mercator for these cases. If `lat_0` is not specified, then 0° (Equator) is assumed in the secant case and ϕ_1 in the tangent case.

Oblique Mercator (Rectified Skew Orthomorphic)	C E I	`+proj=omerc`
		`+k_0=`k_0
		`+lat_0=`ϕ_0
		`+no_rot`
		`+no_uoff`
		`+rot_conv`
		two point
		`+lon_1=`λ_1
		`+lat_1=`ϕ_1
		`+lon_2=`λ_2
		`+lat_2=`ϕ_2
		azimuthal
		`+alpha=`α_c
		`+lonc=`λ_c
			Two means of specify cartographic control are:
			1. two points on the projection centerline (λ_1, ϕ_1) and (λ_2, ϕ_2),
			2. a point of origin at (λ_c, ϕ_0) and an azimuth, measured clockwise from North, of the projection centerline α_c.
			The presence of the `+alpha` option determines which method is used. The projection centerline approximates a geodesic.
			Unless the `+no_rot` option is specified, the coordinates are rotated by α_c (computed internally with the two point method) or by the origin convergence angle when `+rot_conv` is specified. In some cases, an offset in the pre-rotated axis may need to be suppressed with the `+no_uoff` option. The scale factor, k_0, applies to the projection origin.
			Initialization will fail if parameters define a nearly transverse or normal Mercator projection.

* C–Conformal, E–elliptical, I–inverse

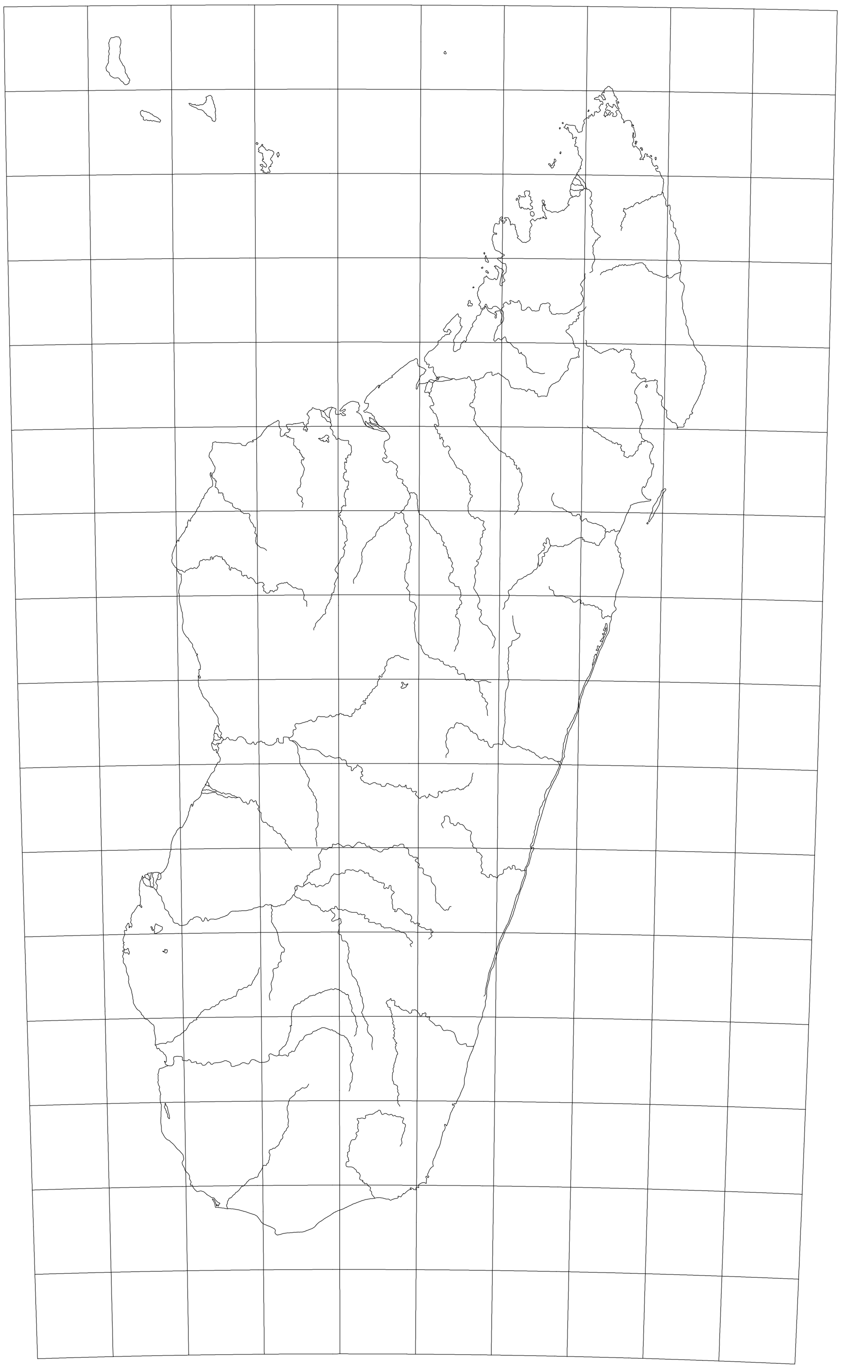


Figure 2: Laborde projection of Madagascar with shorelines and 1°graticule.

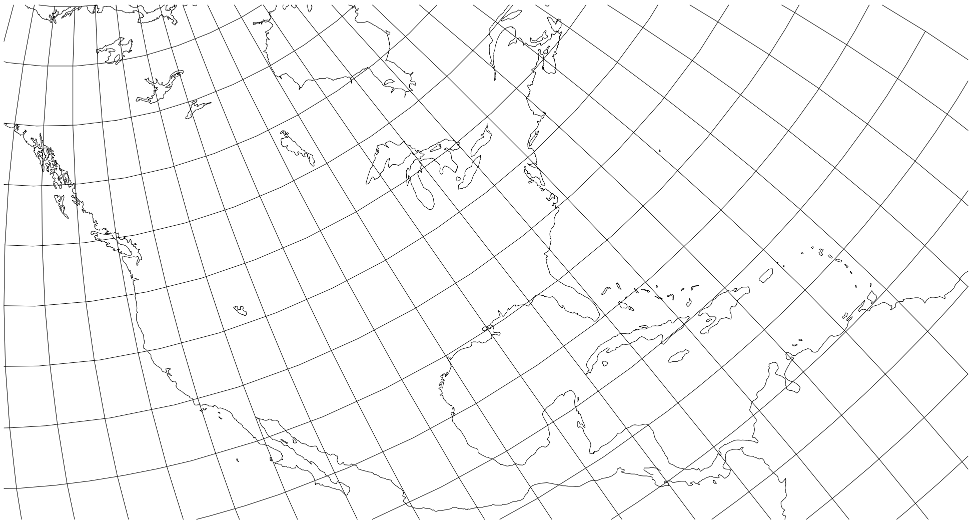
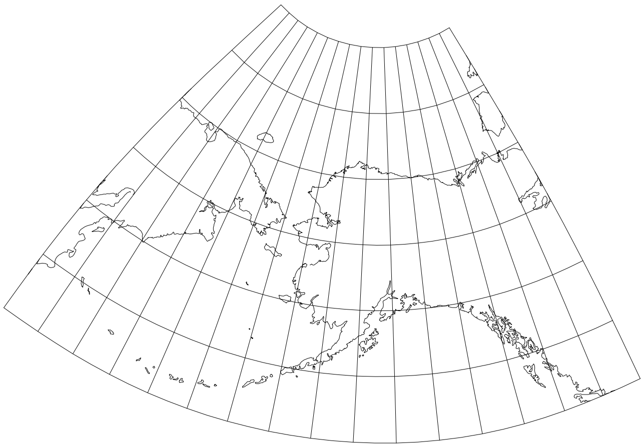
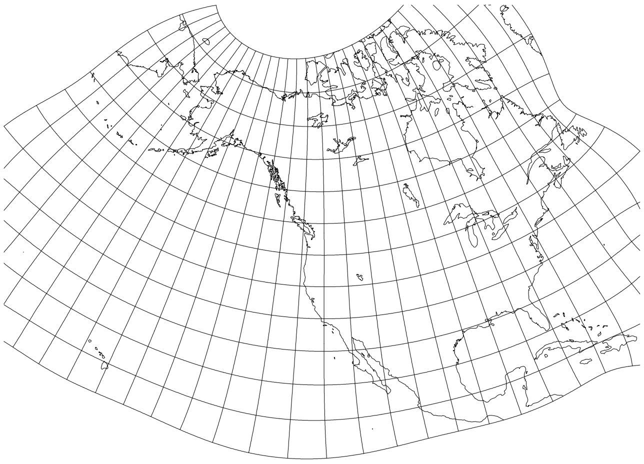


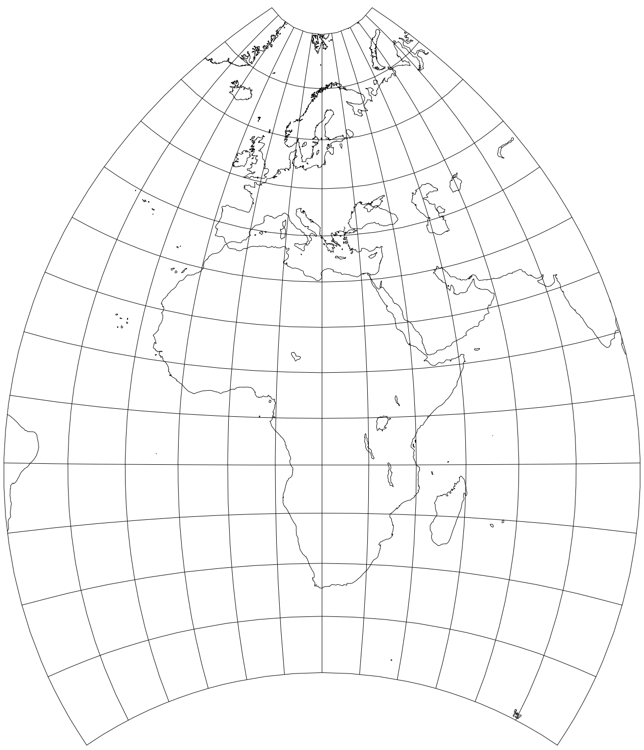
Figure 3: Two Point Equidistant projection, with shorelines and 5° graticule. Central points at Seattle, Washington and Charlotte Amalie, U.S. Virgin Islands (`+proj=tpeqd +lon_1=122d20w +lat_1=47d36n +lon_2=64d54w +lat_2=18d21n`).



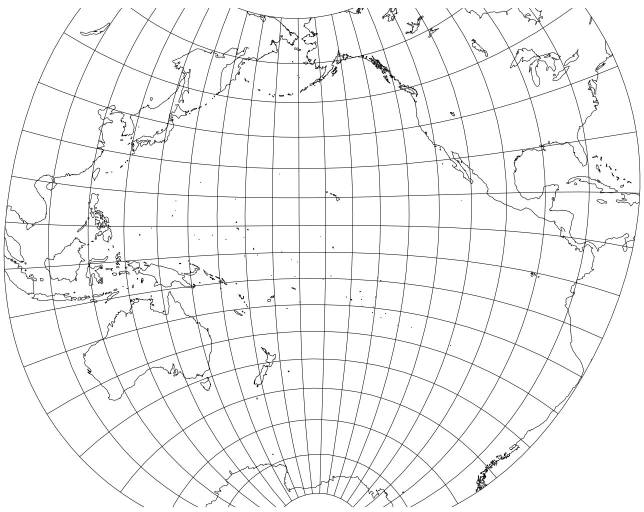
A – Alaska Modified Stereographic
5° graticule (`+proj=alsk`)



B – 50 United States Modified Stereographic
5° graticule (`+proj=gs50`)



C – Miller Oblated Stereographic
10° graticule (`+proj=mill_os`)



D – Lee Oblated Stereographic
10° graticule (`+proj=lee_os`)

Figure 4: Modified Stereographic projections with shorelines and graticules

Programming with the Cartographic Library

Use of cartographic projections in computer applications is varied and potentially complex and, although a program such as **proj** can serve variety of needs, there are many situations where more specialized programs are more appropriate or required. To support alternate applications, the software was developed to be modular and encapsulated so that the application programmer can concentrate efforts on the unique needs of the application and not on the details of cartographic mathematics. This section describes usage of the principle entries of the projection library and Appendix 3 contains a summary of all the entries to procedures of potential programmatic interest.

Basic Usage

A cartographic projection is similar to the standard transcendental functions included in the compilers mathematics library such as `sin(x)` to compute $\sin x$ and `asin(x)` to compute the inverse, $\sin^{-1} x$. But unlike the transcendental functions, the forward, P, and inverse, P^{-1}, cartographic projection functions have a multivariate argument and a bivariate return value:

$$(x, y) = P(\lambda, \phi, \cdots) \quad (1)$$

$$(\lambda, \phi) = P^{-1}(x, y, \cdots) \quad (2)$$

where x and y are the cartesian coordinates, usually in meters, and λ and ϕ are the respective longitude and latitude geographic coordinates in radians. There is always either the Earth's radius, R, or the major ellipsoid major axis, a, and one of the means of specifying ellipsoid shape that are part of the remaining P arguments. The actual number of function arguments is reflected in the tabulation of the cartographic parameters previously described in the user's sections and include such elements as central meridian, standard parallels, false easting and northing, ...

Because of the large number of selectable projections, each with their own special list of arguments, the following method was chosen to simplify the number of library entries needed by the programmer to the following prototypes defined in the header file `projects.h`:

```
PJ *pj_init(int, char **);
UV pj_fwd(UV, PJ *);
UV pj_inv(UV, PJ *);
void pj_free(PJ *);
```

The complexity of this system is not in programmatic usage as described in the following text, but in understanding and properly using the cartographic control parameters.

The procedure `pj_init` must be called first to select and initialize a projection. Parameters for the projection are passed in a manner identical with the normal C program entry point `main`: a count of the number of parameters and a list of pointers to the characters strings containing the parameters. In this case, the parameter strings are those cartographic parameters discussed in the section on using program **proj** and the projection tables. This also includes references to initialization files and the use of the default file.

If the initialization call to `pj_init` fails, then a null or `(PJ *)0` value is returned. Otherwise, `pj_init` returns a pointer that is used as an argument with the forward, `pj_fwd`, and inverse, `pj_inv`, projection functions. The first argument argument to the forward and inverse projection function and the function return is a type declared (in the header file `projects.h`) as:

```
typedef struct { double u, v; } UV;
```

where `u` and `v` respective x and y cartesian coordinates or respective longitude, λ, and latitude, ϕ, geographic coordinates [1] in radians. If either the forward or inverse function fail to perform a conversion, both `u` and `v` in the returned structure are set to `HUGE_VAL` as defined in the `math.h` header file.

Two additional notes should be made about the header file `projects.h`: it contains includes to the system header files `stdlib.h` and `math.h`, and several predefined constants such as multipliers `DEG_TO_RAD` and `RAD_TO_DEG` to respectively convert degrees to and from radians.

To illustrate usage, the following is an example of a filter procedure, `example1.c`, designed to convert input pairs of decimal latitude and longitude values in decimal degrees to corresponding cartesian coordinates using the Polyconic projection with a central meridian of 90°W and the Clarke 1866 ellipsoid:

```
#include <stdio.h>
#include <projects.h>
main(int argc, char **argv) {
    static char *parms[] = {
       "proj=poly",
       "ellps=clrk66",
       "lon_0=90W",
       "no_defs"
    };
    PJ *ref;
    UV data;

    if ( ! (ref = pj_init(sizeof(parms)/sizeof(char *), parms)) ) {
       fprintf(stderr, "Projection initialization failed\n");
       exit(1);
    }
    while (scanf("%lf %lf", &data.v, &data.u) == 2) {
       data.u *= DEG_TO_RAD;
       data.v *= DEG_TO_RAD;
       data = pj_fwd(data, ref);
       if (data.u != HUGE_VAL)
          printf("%.3f\t%.3f\n", data.u, data.v);
       else
          printf("data conversion error\n");
    }
    exit(0);
}
```

Assuming that the header file has been installed in `/usr/local/include` and the projection library in `/usr/local/lib`, then the example can be compiled and loaded by:

```
# cc -I/usr/local/include example1.c -L/usr/local/lib -lproj -lm
```

To test the program, the script

```
# a.out <<EOF
0 -90
```

[1] An argument can be made that giving both coordinates systems the same type name is bad style, but the author has found through experience that this method is generally much more convenient because the functions are often used interchangeably.

```
33 -95
77 -86
EOF
```

should give the results:

```
0.000   0.000
-467100.408     3663659.262
100412.759      8553464.807
```

The previous example can be expanded to create a more flexible program with runtime selection of projection parameters by removing the `parms` declaration and initialization, and substituting the `pj_init` parameters with the arguments from `main` entry:

```
if ( ! (ref = pj_init(--argc, ++argv)) ) {
```

Recompiling the program and executing it as:

```
# a.out proj=poly ellps=clrk66 lon_0=-90 no_defs
```

will give the same results as the original program. The use of `+` parameter prefix as in the case with program **proj** is only to flag the runline values as non-files, much in the same manner that `-` is uses to flag options. In this case, runline files are not part of the program, so use of `+` is not needed.

When executing `pj_init` the projection system allocates memory for the structure `PJ`. This allocation is complex and consists of two or more memory allocations to assign substructures referenced within `PJ`. Although the previous examples did not require its usage, certain applications are foreseen where repeated calls to `pj_init` are made to re-initialize a projection with different parameters. The function `pj_free` should be used to ensure proper memory deallocation of a previously initialized `PJ` pointer when the process has no further need for the structure.

Limiting Selection of Projections

Many applications will only need a small subset of the projections contained in the library `libproj.a`, but unless some action is taken, all of the projections will be linked into the final process. This is not a problem unless the memory requirements of the application are to be kept small or access to projections is to be restricted.

If there is a need to limit the number of projections, a simple two-step process needs to followed. First creat a header file, `my_list.h` for example, that contains a list of macro calls `PROJ_HEAD(`*id*, *text*`)`, one for each projection to be part of the application program. Argument *id* is the acronym of the projection and argument *text* is the ascii string describing the program (what appears after the colon in **proj**'s `-l` execution. The header file, `nad_list`, for program **nad2nad** is a an example:

```
/* projection list for program nad2nad */
PROJ_HEAD(lcc, "Lambert Conformal Conic")
PROJ_HEAD(omerc, "Oblique Mercator")
PROJ_HEAD(poly, "Polyconic (American)")
PROJ_HEAD(tmerc, "Transverse Mercator")
PROJ_HEAD(utm, "Universal Transverse Mercator (UTM)")
```

An easy way to create this list is to copy and edit the file `pj_list.h` in the source distribution, which contains the entire listing of available projections, and edit out of the copy all lines of unwanted projections.

Next, in one of the program code modules that includes the header file `projects.h`, preceed the `include` statement with:

```
#define PJ_LIST_H "my_list.h"
```

Be careful to only put this include in only one of the code modules because this define action causes the initialization of the global `pj_list` and multiple initializations will cause havoc with the linker.

When no action is taken to limit the number of linked projections, the module `pj_list.o` from the library is used which causes linkage of all distributed projections. The savings in program size can be considerable. In the case of program **nad2nad**, the use of the above process yields a program of about 48kbytes while ignoring the process creates a program of about 154kbytes—more than three times larger.

Error Handling

Error handling in the projection system is performed in much the same manner as the standard ANSI C library procedures. In cases where a functional value is returned, the returned value assumes a special state such as a null pointer or double precision `HUGE_VAL`. The system also sets a global type `int` value `pj_errno` to a non-zero value indicating the cause of the error. Although similar to the ANSI standard's `errno`, it differs in two properties: it is never used as a macro and it, as well as `errno`, is reset to zero at each execution of `pj_init`, `pj_fwd` and `pj_inv`.

To provide users with an indication of the type of error encountered, the function

```
char *pj_strerrno(int pj_errno)
```

may be used to obtain a string for display. Similar to the ANSI C function `strerror`, the string pointed to cannot be modified.

The projection system uses negative values for `pj_errno` for all errors detected by projection system tests. If C library system errors occur during execution of the projection system, thus causing `errno` to return a positive value, and the projection system otherwise does not detect an error, the value of `pj_errno` will be set to `errno` and the functional results will be set to the error values. In these cases, the string pointer returned by the function `pj_strerrno` will be that of the C library function `strerror`.

More Complete Program Example

With the same basic criteria of `example1.c` program with the added restriction that only Transverse Mercator and Polyconic projections are to be computed, DMS input data and better error diagnostics of the initialization, the following `example2.c` program is written:

```
#include <stdio.h>
#define PJ_LIST_H "examp2.h"
#include <projects.h>
main(int argc, char **argv) {
    PJ *ref;
    UV data;
    char lat[40], lon[40];

    if ( ! (ref = pj_init(argc, argv)) ) {
       fprintf(stderr, "Projection initialization failed\n"
          "because: %s\n", pj_strerrno(pj_errno));
       exit(1);
    }
    while (scanf("%39s %39s", lat, lon) == 2) {
       data.u = dmstor(lon, 0);
       data.v = dmstor(lat, 0);
```

```
        data = pj_fwd(data, ref);
        if (data.u != HUGE_VAL)
            printf("%.3f\t%.3f\n", data.u, data.v);
        else
            printf("*\t*\n");
    }
    exit(0);
}
```

and where header file `examp2.h` contains:

```
PROJ_HEAD(poly, "Polyconic (American)")
PROJ_HEAD(tmerc, "Transverse Mercator")
```

Compiling and linking the program in the same manner as the first example and executing with the following script:

```
# a.out proj=tmerc ellips=clrk66 lon_0=90w <<EOF
33.3 -90.55
44d15'7.5 87d10'15.4w
EOF
```

should give the results:

```
-51226.063      3685962.942
225953.937      4905510.287
```

The resulting total size of this program with limited projections was 28,712 bytes versus 117,988 bytes for the first example. Of course, these size values vary with different host systems but it does give an indication of possible memory savings when limiting the number of projection procedures linked into the program.

Library Lists

Program **proj** as well as the previous examples are designed as filter programs executed from the run-line and not interactive programs with user dialog capability. To fully discuss mechanisms to construct interactive programs using the cartographic procedures is beyond the scope of this report, but description of some of the projection system internals can be useful in interactive applications.

There three option list structures in the system described in the header file `projects.h`:

```
struct PJ_LIST {
    char    *id;         /* projection keyword */
    void    *(*proj)(); /* projection entry point */
    char    *const*name;     /* basic projection full name */
} pj_list[];
struct PJ_ELLPS {
    char    *id;    /* ellipse keyword name */
    char    *major; /* a= value */
    char    *ell;   /* elliptical parameter */
    char    *name;  /* comments */
};
#ifndef PJ_ELLPS__
extern struct PJ_ELLPS pj_ellps[];
#endif
struct PJ_UNITS {
    char    *id;    /* units keyword */
    char    *to_meter;  /* multiply by value to get meters */
```

```
    char    *name;  /* comments */
};
#ifndef PJ_UNITS__
extern struct PJ_UNITS pj_units[];
#endif
```

The first, `PJ_LIST`, simplified for clarity here, has already been described when discussing the alteration of the list of projections to be linked into a program. But it, as well the others, can be used in interactive option displays (program **proj** performs a display of these lists through the `-lp`, `-le` and `-lu` run-line options).

In each list, the `id` pointer refers to the argument value for the `proj=`, `ellps=` and `units=` initialization parameters and the associated `name` points to a more human readable string describing the entry. In an interactive program, the `name` entry can be displayed in a scrolled list and, maintaining an equivalence of indicies, use the index returned by user selection to generate the string needed by the argument list for `pj_init`.

Matrix Datum Conversion.

The matrix method of datum conversion is the use of a two dimensional matrix of correction values to be added to an input of one datum to determine the value in another datum. The row-column interval of the matrix is constant and sufficiently spaced to allow semi-linear interpolation of correction values not located on a node by the bivariate four-point formula (Eqn. 25.2.66, p. 882, Abramowitz and Stegun, 1965):

$$\begin{aligned}
f(u_i + ph, v_j + qk) &= (1-p)(1-q)f_{i,j} + p(1-q)f_{i+1,j} && (3)\\
&\quad +q(1-p)f_{i,j+1} + pqf_{i+1,j+1} + O(h^2) && (4)\\
p &= (u - u_i)/h && (5)\\
q &= (v - v_j)/k && (6)\\
h &= u_{i+1} - u_i && (7)\\
k &= v_{i+1} - v_i && (8)
\end{aligned}$$

In the application of correcting NAD27 datum to NAD83 datum the respective u and v are longitude and latitude and f is a value to be added to the NAD27 coordinates in order to convert to NAD83. The inverse correction is determined by simple, direct iteration of detemining a point that produces a corrected value.

Usage of this system is similar to the usage of the projection system: creating and initializing a control structure and subsequent calls to the correction procedure. Prototypes defined in the header file `projects.h` are:

```
struct CTABLE *nad_init(char *)
UV nad_cvt(UV, int, struct CTABLE *)
void nad_free(struct CTABLE *)
```

Execution of `nad_init` with a string argument defining the name of a correction matrix file covering the region of interest will create and return a pointer to the control structure for this region. Pathing for this file follows the same rules as the projection default and initialization files with the added factor of the directory `nad2783`. If the initialization fails, a null pointer is returned.

Procedure `nad_cvt` returns the geographic coordinates of the first argument as defined by the `CTABLE` structure pointed to by the third argument. If the second argument is non-zero, the inverse correction is made, otherwise the forward correction. When coordinates are outside the region defined by the `CTABLE` structure, `HUGE_VAL` is returned. When doing inverse correction it is possible to move outside the region near the boundary, thus returning `HUGE_VAL`, even though the argument point is within the region.

The procedure `nad_free` closes the structure `CTABLE` and returns allocated memory to the system. When creating a `CTABLE` structure, the correction matrix is read into memory and a considerable increase in program memory requirements may be expected.

Projection Approximations

Cartographic projections can be computationally complex and some uses will increase this complexity by requiring multiple projection evaluations and other computations for each point processed. Thus, when a large number of points are to be processed, a considerable amoung of computer processing will be used in the transformation process. Although costs of computing have declined and computer speed has substantially increased, the geometric increase in the volume of data as well as the need for fast processing (often for interactive graphics) encourages the use of effective alternatives to the use of the analytic projection procedures.

Snyder (1985) reviews projection approximations but limits discussion to power series developed by either Taylor series expansions or least-squares methods. These techniques often work, but it is desirable to follow more traditional function approximation methods that are based upon the premise of minimizing the maximum error of the approximation: MINIMAX. True MINIMAX approximations are difficult to determine, but there is a simple and easily applied method that nearly reaches this goal.

Chebyshev Approximation

The approximation method used in this system is the Chebyshev method because of its property of error determination, its near MINIMAX characteristics and the ease in determining its coefficients. Application of this method to univariate functions is well known, but neither theory nor application references for multivariate applications have been located. However, practice has shown that the following intuitive expansion of the Chebyshev method can work for bivariate cartographic applications and most of the procedures used in this system were developed by adaptation of the univariate procedures described in *Numerical Recipes in C* (Press, *et al.*, 1988).

In the univariate case, a function, $f(u)$, may be approximated over the argument inverval $-1 \leq u \leq 1$ by:

$$f(u) \approx \sum_{i=0}^{N}{}' c_i T_i(u) \tag{9}$$

using Fox and Parker (1968) notation where the prime indicates that the c_0 term is halved at evaluation and where $T_i(u)$ is the Chebyshev polynomial of degree n. The c_0 coefficients are determined by:

$$c_n = \frac{2}{N+1} \sum_{k=0}^{N} f(u_k) \cos(n u_k) \tag{10}$$

where

$$u_k = \cos\left(\frac{2k+1}{N+1} \cdot \frac{\pi}{2}\right) \tag{11}$$

Because $|T_n(u)| \leq 1$ for $-1 \leq u \leq 1$, and (9) is exact for $N = \infty$, the accuracy of the approximation of a non-infinite N can be assessed by examination of the coefficients c_n. When the value of the coefficients converge to zero with increasing

n, a value of N can be selected for an approximation with the maximum error, $|E|$, of this truncation being:

$$|E| \leq \sum_{n=N+1}^{\infty} |c_n|. \tag{12}$$

In practice, the value of N is set to a value expected to be considerably higher than needed and then adjusted to a lower value, N', such that:

$$|E| \leq \sum_{n=N'+1}^{N} |c_n|. \tag{13}$$

where $|E|$ is the required precision of the application.

To apply the Chebyshev method to a bivariate expression, (9) is rewritten as:

$$f(u,v) \approx {\sum_{i=0}^{N}}' \left[{\sum_{j=0}^{M}}' c_{i,j} T_j(v) \right] T_i(u) \tag{14}$$

The braces are used to emphasize the order of evaluation. Similarly, the coefficients are determined by:

$$c_{n,m} = \frac{2}{N+1} \sum_{k=1}^{N} \left[\frac{2}{M+1} \sum_{l=0}^{M} f(u_k, v_l) \cos(mv_l) \right] \cos(n, u_k) \tag{15}$$

where u_k is the same as (11) and:

$$v_l = \cos\left(\frac{2l+1}{M+1} \cdot \frac{\pi}{2} \right) \tag{16}$$

The coefficients, $p_{i,j}$, for the bivariate power series

$$f(u,v) \approx \sum_{i=0}^{N} \sum_{j=0}^{M} p_{i,j} u^i v^j \tag{17}$$

can be derived from the Chebyshev series by adaptation of the univatiate conversion described by Press *et al.* (1988). Loss of computational precision can occur with increasing N or M and it is not recommended when the sum of the powers of any coefficient exceeds 6 or 7. But when the power series can be used, it is the fasted method.

Cartographic Application

To apply Chebyshev approximations to cartographic transformation applications, the following **proj** library user entries are available:

```
Tseries *mk_cheby(UV a, UV b, double res, UV *resid,
   UV (*func)(UV), int NU, int NV, int pwr)
UV biveval(UV val, Tseries *coefs)
```

The procedure `mk_cheby` determines the two sets of Chebyshev coefficients, one for each axis, that are stored in the the structure pointed to by `Tcheby`, for the function defined by `func` over the argument range defined by `a` and `b` that specify the respective lower and upper range limits input arguments. Argument `res` defines the precision of the approximation such that the maximum absolute error must be ≤`res`. The values returned in the address pointed to by `resid` are the sums of the absolute values of the discarded coefficients. If `mk_cheby` returns a null pointer, an error was encountered. If the value of `resid.u` is less than zero, adjustment criteria for N were not met and the approximation may not meet error criteria—this is a warning.

The `mk_cheby` arguments `NU` and `NV` are the initial number of coefficients to be determined in the respective u, v axis (note that $N =$ `NU` $- 1$ and $M =$ `NV` $- 1$). Values of `NU=NV=15` are adequate for most applications.

When `pwr` is not zero, the power coefficients to be returned in structure `Tseries`, otherwise Chebyshev coefficients are returned.

After a successful execution of `mk_cheby`, transformations may be performed by `biveval` in a manner similar to `pj_fwd` or `pj_inv`. Evaluation of the Chebyshev approximation is performed by a bivariate adaptation of Clenshaw's method and Horner's method method is used for the power series.

The returned structure, `Tseries`, is declared in the header file `projects.h` as:

```
typedef struct {  /* Chebyshev or Power series structure */
    UV a, b;      /* power series range for evaluation */
                  /* or Chebyshev argument shift/scaling */
    struct PW_COEF { /* row coefficient structure */
       int m;     /* number of c coefficients (=0 for none) */
       double *c  /* power coefficients */
    } *cu, *cv;
    int mu, mv;   /* maximum cu and cv index (+1 for count) */
    int power;    /* != 0 if power series, else Chebyshev */
} Tseries;
```

The user should examine the row indicies and maximum column counts to ensure that the values of `NU` and `NV` were sufficiently larger (say a factor of 2) to validate the residual error estimates.

A simple example of using the approximation procedure is determining the approximation coefficients for converting geographic coordinates to the Massachusetts Mainland Zone SPCS cartesian coordinates. In this case, the geographic range is between 73.5°W and 69.5°W longitude and 41°N and 43°N latitude and the output is to be in U.S. feet and accurate to 0.01 foot (or $|E| \leq 0.005$ft).

```
#include <stdio.h>
#include <projects.h>
static PJ *P;
static UV func(UV arg) {  /* function for mk_cheby */
    return (pj_fwd(arg, P));
}
main() {
    char *largv[] = {
       "units=us-ft",
       "init=nad27:2001",
    };
    UV a, b, sums;
    int NU, NV, pwr;
    Tseries *T;
    extern void pr_series(Tseries *, FILE *, char *);

    /* initialize projection */
    if (!(P = pj_init(sizeof(largv)/sizeof(char *), largv))) {
       printf("failed: %s\n", pj_strerrno(pj_errno));
       exit(1);
    }
    /* set limits */
    a.u = -73.5 * DEG_TO_RAD;
    b.u = -69.5 * DEG_TO_RAD;
    a.v = 41. * DEG_TO_RAD;
    b.v = 43. * DEG_TO_RAD;
```

```
        NU = NV = 15;
        pwr = 0;
        /* generate approximation polynomial */
        if (!(T = mk_cheby(a, b, .005, &sums, func, NU, NV, pwr))) {
           printf("failed cheby\n");
           exit(1);
        }
        printf("est. max error: %g %g\n", sums.u, sums.v);
        pr_series(T, stdout, "%.3f");
    }
```

Output of `printf` and `pr_series` procedure is:

```
est. max error: 0.00039222 0.00292703
u: 4
0 1 2400000.000
1 4 1087200.999 -8544.032 -0.249 -0.108
3 2 -24.907 0.196
v: 5
0 5 1470366.610 728721.820 21.199 9.207 0.018
2 2 6373.251 -50.086
4 1 -0.073
```

Several items should be noted in this example:

- Function `pr_series` is a utility supplied with the distribution but not part of the library which prints coefficients of the `Tseries` structure on the specified stream and format. Output consists of the tag `u:` and `v:` followed by the number of rows for respective u, v axis, followed by lines with row index, number of columns and column coefficients. Rows with all zero coefficients are omitted.

  By selection of the longitude range centered about the central meridian of the projection, use of the symmetry in the u axis created even and odd series and thus optimizing evaluation,

- initial number of coefficients selected for each axis is adequate,

- because the error tends to decrease in order of magnitude steps, 0.001 foot accuracy could probably be achieved with only a couple additional terms in the v conversion.

To determine the power series, change `pwr=1` and the format for `pr_series` to `%.15g` and the following output will result:

```
sums: 0.00039222 0.00292703
u: 4
0 4 30198167.5141474 -20046865.4522086 6326812.52401693 -2903510.90579901
1 4 17662488.1318286 -12779414.4082363 5069925.25030325 -2326698.19122364
2 2 -7279001.83771805 3948519.25927063
3 2 -1944317.40964669 1054701.579871
v: 5
0 5 -3000106.82619256 17817210.2348729 -5076400.10623976 1216337.88760375
  766389.396083807
1 2 20143924.5562269 -11756157.4427093
2 2 6845916.58117997 -4710337.09621368
3 1 -981757.47335366
4 1 -196680.278771301
```

Computational efficiency is lost due to shifting of the longitude range so that the series are no longer even and odd. To compensate for this and produce a more comprehensible set of coefficients, the following procedure modifications can be made:

```
    ...
static UV base;
static UV func(UV arg) {  /* function for mk_cheby */
    arg.u += base.u;
    arg.v += base.v;
    return (pj_fwd(arg, P));
}
    ...
    base.u = -71.5 * DEG_TO_RAD;
    base.v =  41.0 * DEG_TO_RAD;
    a.u = -2. * DEG_TO_RAD;
    b.u =  2 * DEG_TO_RAD;
    a.v =  0. * DEG_TO_RAD;
    b.v =  2. * DEG_TO_RAD;
    ...
```

This results in the output:

```
sums: 0.00039222 0.00292703
u: 4
0 1 600000
1 4 15818858.7563991 -14025128.2322332 75074.3214800214 -2326698.19199796
3 2 -1189588.7866301 1054701.5798669
v: 5
0 5 -0.000122078398817393 20879148.4820533 -110587.906504021 3410004.88431155
  766389.490983865
2 2 5312988.10592422 -4710337.09621398
4 1 -196680.278779712
```

that can be readily editted into the following procedure:

```
typedef struct {double u, v;} UV;
UV /* forward projection of Mass. Mainland Zone, NAD 1927 */
mass_main27(UV in) {
    double u, v, u2;

    u = in.u + 1.2479104151759456475004; /* 71.5 */
    v = in.v - 0.7155849933176751265387; /* 41 */
    u2 = u * u;
    in.u = 600000. + u * (15818858.7563991 + v * (-14025128.2
       + v * (75074. + v * -2326698.))
       + u2 * (-1189589. + v * 1054702.));
    in.v = v * (20879148.48 + v * (-110588. + v * (3410005.
       + v * 766389.))) + u2 * ( 5312988. + v * -4710337.
       + u2 * -196680.);
    return in;
}
```

At this point, the speed of execution of these approximations can be compared to the analytic projection function. Although performance will vary with complexity of the projection, precision of approximation, hardware, operating system and compiler, the performance shown in Table 5 is indicative of what may be expected. In this case, the use of the Chebyshev polynomial is nearly three times faster than the analytic evaluation.

Table 5: Performance characteristics of approximation methods applied to forward projection of Massachussetts Mainland Zone on an Intel 66Mhz i486DX2 processor and a UNIX operating system.

Method	Speed μsec.	Perf. Incr.
Analytic projection function	117	1.0
Chebyshev series (`biveval`)	40	2.9
Simple power series (`biveval`)	28	4.2
Modified power series procedure	8	14.6

Appendix 1—Summary of program proj commands

This is a short summary of the usage of program **proj**. Much of this material is repeated in the manual file `proj.1` included with **proj** distribution and that may be made available as an on-line resource.

Execution of **proj** is performed as:

`proj [-`*control*`] [+`*control*`] [`*files*`]`

On Unix systems, the program name `invproj` may be used to select inverse projection mode. Input data *files* may be specified on the run-line and are processed in a left to right order and a ␣-␣ may be used to indicate data to be processed from *stdin*. If no data files are specified, input is assumed to be from *stdin*.

The `-`*control* run-line parameters are restricted to controlling the nature of data input and output and basic selections of information to be computed. The following run line `-`*control* parameters can appear in any order:

`-I` — Select inverse projection computations where input is cartesian coordinates and output is geographic coordinates. When not specified and program name does not start with `inv`, forward computations are performed where input is geographic and output is cartesian. Use is redundant when `invproj` is used.

`-l[p|P|e|u] -l=`*id* — This option causes an output listing of the current projections (`-lp`), ellipsoids (`-le`) and unit conversions (`-lu`) supported by the program. Option `-lP` produces an expanded listing with supplementary information about each projection and `-l=`*id* outputs the same output for an individual projection *id*.

`-b` — Special option for binary coordinate data input and output through standard input and standard output. Data is assumed to be in system type *double* floating point words. This option is useful when **proj** is a *son* process and allows bypassing formatting operations.

`-i` — Selects binary input only (see `-b` option).

`-o` — Selects binary output only (see `-b` option).

`-t`*a* — *A* specifies a character employed as the first character to denote a control line to be passed through without processing. This option applicable to ASCII input only. (`#` is the default value).

`-e␣`*string* — *String* is an arbitrary string to be output if an error is detected during data transformations. The default value is the string: `*\t*`. Note that if the options `-b`, `-i` or `-o` are employed, an error is indicated by a system defined `HUGE_VAL` output for both values.

`-r` — This options reverses the order of the expected input from longitude-latitude or x-y to latitude-longitude or y-x.

`-s` — This options reverses the order of the output from x–y or longitude-latitude to y–x or latitude-longitude.

`-m␣`*mult* — The cartesian data may be scaled by the *mult* parameter. When processing data in a forward projection mode the cartesian output values are multiplied by *mult* otherwise the input cartesian values are divided by *mult* before inverse projection. If the first two characters of *mult* are `1/` or `1:` then the reciprocal value of *mult* is employed.

`-f␣`*format* — *Format* is a *printf*(3) format string to control the form of the output values. For inverse projections, the output will be in degrees when this option is employed. If a format is specified for inverse projection the output data will be in decimal degrees. The default format is `%.2f` for forward projection and DMS for inverse.

`-[w|W]`*n* — *N* is the number of significant fractional digits to employ for seconds output (when the option is not specified, `-w3` is assumed). When `-W` is employed the fields will be constant width with leading zeroes.

`-v` — This option serves as a diagnostic to display all parameters used to initialize a projection. Principle usage is to identify misspelled or inappropriate user parameters as well as functioning of the initializing selection or default options file. Parameters entered but not used are also identified.

`-E` — When this option is selected, the input coordinates will be copied to the output stream prior to the printing of the converted results. Should not be used with `-o`, `-i` or `-b`.

`-S` — Usage of this option causes computation and print of scaling and distortion characteristics of the projected point. Output consists of *h*, *k*, *s*, *omega*, *a* and *b* enclosed in angle brackets, `< >`.

`-V` — This option provides a more detailed and annotated analysis than that provided by the `-S` option. In addition, the user can override the forward-inverse mode of **proj** with the input line's first character either a `i` or `I` for inverse or `f` or `F` for forward. Coordinates must be as longitude-latitude or *x*-*y* order and binary I/O is not allowed. Information after the coordinates is passed on as comments and a line beginning with `#` is ignored.

`-T␣`*ulow,uhi,vlow,vhi,res[,umax,vmax]* — This option creates an ASCII output structure of coefficients and control data for projection conversion using Chebyshev polynomials. Arguments *ulow-uhi* are longitude or x data ranges and *vlow-vhi* are latitude or y data ranges depending on respective forward or inverse projection mode.

The `+`*control* run-line arguments are associated with cartographic parameters and usage varies with projection selected and reference should be made to specific projection documentation. Except for `+init`, these control parameters, with or without the `+`, may also be used in the initialization file referenced by `+init` or the defaults file. The options are processed in left to right order from the run-line followed by processing entries in optionally selected initialization file and defaults file. Reentry of an option is ignored with the first occurrence assumed to be the desired value.

`+proj=`*name* — is **always** required for selection of the cartographic transformation function and where *name* is an acronym for the desired projection.

`+init=`*file:key* — names a *file* containing cartographic control parameters associated with the keyword *key*.

`+R=`*R* — specifies that the projection should be computed as a spherical Earth with radius *R*.

`+ellps=`*acronym* — The `+ellps` option allows selection of standard, predefined ellipsoid figures. For spherical only projections, the major axis is used as the radius.

`+a=`*a* — specifies an elliptical Earth's major axis *a*.

`+es=`e^2 — defines the elliptical Earth's squared eccentricity. Optionally, either `+b=`b, `+e=`e, `+rf=`$1/f$ or `+f=`f may be used where b, e and f are respective minor axis, eccentricity and flattening.

`+R_A` — must be used with elliptical Earth parameters. It determines that spherical computations be used with the radius of a sphere that has a surface area equivalent to the selected ellipsoid. `+R_V` can be used in a similar manner for sphere radius of an ellipse with equivalent volume.

`+R_a` — must be used with elliptical Earth parameters. Spherical radius of the arithmetic mean of the major and minor axis is used. `+R_g` and `+R_h` can be used for equivalent geometric and harmonic means of major and minor axis.

`+R_lat_a=`ϕ — must be used with elliptical Earth parameters. Spherical radius of the arithmetic mean of the principle radii of the ellipsoid at latitude ϕ is used. `+R_lat_g=`ϕ can be used for equivalent geometric mean of the principle radii.

`+x_0=`x_0 — false easting; added to x value of the cartesian coordinate. Used in grid systems to avoid negative grid coordinates.

`+y_0=`y_0 — false northing; added to y value of the cartesian coordinate. See `-x_0`.

`+lon_0=`λ_0 — central meridian. Along with `+lat_0`, normally determines the geographic origin of the projection.

`+lat_0=`ϕ_0 — central parallel. See `+lon_0`.

`+units=`*name* — selects conversion of cartesian values to units specified by *name*. When used, other `+` metric parameters must be in meters.

`+geoc` — data geographic coordinates are to be treated as geocentric when this option specified.

`+over` — inhibit reduction of input longitude values to a range within $\pm 180°$ of the central meridian.

Site installations usually have a default directory path in **proj** to indicate the location of unqualified initialization file names and the projection default file `proj_def.dat`. The environment parameter `PROJ_LIB` can be used to define a new directory for this path.

Appendix 2—Summary of program nad2nad commands

This is a summary of the usage of program **nad2nad**. Much of this material is repeated in the manual file `nad2nad.1` included with **proj** distribution and may be available as an on-line resource.

Execution of **nad2nad** is performed as:

`nad2nad [-`*control*`] [`*files*`]`

Input data *files* may be specified on the run-line and are processed in a left to right order and a ␣-␣ may be used to indicate data to be processed from *stdin*. If no data files are specified, input is assumed to be from *stdin*.

The `-`*control* run-line parameters control the nature of data input and output and basic selections of how information is to be processed. The following run line `-`*control* parameters can appear in any order:

`-i|o`␣*option*[*,option* ...] — specify the nature of the input (`-i`) and output (`-o`) data and how it is to be processed. The following options are applicable to both:

`27|83` — data datum year. If omitted, 27 assumed.

`utm=`*zone* — data in UTM grid coordinates for zone number *zone*.

`spcs=`*zone* — data in SPCS grid coordinates for State zone number *zone*.

`hp=`*zone* — data in high precision grid coordinates in one of the following zones:

		Extent			
Region	*zone*	East	West	South	North
Florida	`FL`	88° W	80° W	24° N	32° N
Maryland	`MD`	80° W	74° W	37° N	41° N
Tennessee	`TN`	91° W	81° W	34° N	38° N
Washington–Oregon	`WO`	125° W	116° W	41° N	50° N
Wisconsin	`WI`	94° W	88° W	42° N	48° N

Must be used with `83` datum.

`feet` — data units are in U.S. Surveyor's feet. This is allowed only when the `spcs` option has been used. Meters are used for all other cartesian data.

`bin` — data are in binary form.

`rev` — data are in reverse order: either latitude–longitude or y–x.

`-t`*a* — *A* specifies a character employed as the first character to denote a control line to be passed through without processing. This option applicable to ASCII input only. (`#` is the default value).

`-e`␣*string* — *String* is an arbitrary string to be output if an error is detected during data transformations. The default value is the string: `*\t*`. Note that if the options `-b`, `-i` or `-o` are employed, an error is indicated by a system defined `HUGE_VAL` output for both values.

`-r`␣*region* — must be given when the values `27` and `83` are different for the `-i` and `-o` options. *Region* is the name of the correction matrix file for the transformation between NAD datums and must be one of the following:

		Extent			
Region	*region*	East	West	South	North
Conterminous U.S.	`conus`	131° W	63° W	20° N	50° N
Alaska	`alaska`	166° E	128° W	46° N	77° N
Hawaii	`hawaii`	161° W	154° W	18° N	23° N
Puerto Rico and Virgin Islands	`prvi`	68° W	64° W	17° N	19° N
St. George Is., AK	stgeorge	171° W	169° W	56° N	57° N
St. Lawrence Is., AK	srlrnc	172° W	68° W	62° N	64° N
St. Paul Is., AK	stpaul	171° W	169° W	57° N	58° N

`-f`␣*format* — *Format* is a *printf*(3) format string to control the form of the output values. For inverse projections, the output will be in degrees when this option is employed. If a format is specified for inverse projection the output data will be in decimal degrees. The default format is `%.2f` for forward projection and DMS for inverse.

`-[w|W]`*n* — *N* is the number of significant fractional digits to employ for seconds output (when the option is not specified, `-w3` is assumed). When `-W` is employed the fields will be constant width with leading zeroes.

`-E` — When this option is selected, the input coordinates will be copied to the output stream prior to the printing of the converted results. Should not be used with binary I/O.

Site installations usually have a default directory path in **nad2nad** to indicate the location of conversion matrix directory `nad2783`. The environment parameter `PROJ_LIB` can be used to define a new path for this directory.

Appendix 3—Projection Library Entries

This is a summary of basic programmatic entries to the cartographic projection library, `libproj.a`. Online source of this material may be available as the **man** file `pj_init.3` distributed with the system sources.

```
#include <projects.h>
PJ *pj_init(int argc, char **argv)
UV pj_fwd(UV val, PJ *proj)
UV pj_inv(UV val, PJ *proj)
void pj_free(PJ *proj)
struct CTABLE *nad_init(char *name)
UV nad_cvt(UV val, int inverse, struct CTABLE *ctable)
void nad_free(struct CTABLE *ctable)
double dmstor(char *str, char **rstr)
void set_rtodms(int frac, int fixed)
char *rtodms(char *str, double rad, int pos, int neg)
char *pj_strerrno(int errnum)
```

Procedure `pj_init` selects and initializes a cartographic projection with its argument control parameters. `Argc` is the number of elements in the array of cartographic control string pointers `argv` that each contain individual control keyword assignments (e.g. `+` **proj** arguments). The list must contain at least the `proj=`*projection* and Earth's radius or elliptical parameters. If the initialization of the projection is successful a valid address is returned otherwise a NULL value.

Once initialization is performed either forward or inverse projections can be performed with the returned value of `pj_init` used as the argument `proj`. Some projections do not have inverse capability; a state that can be determined by `proj->inv==0`. The type `UV` is a structure

```
typedef struct { double u, v; } UV;
```

where the values `u` and `v` contain respective longitude and latitude, in radians, or x and y. If a projection operation fails, both elements of the returned `UV` value are set to `HUGE_VAL` (defined in `math.h`).

Memory associated with the projection initialization pointer, `proj` may be freed with `pj_free`.

Procedure `nad_init` returns a pointer to a control structure that is used to convert geographic coordinates between datums by means of a bivatiate matrix stored in the file named in `name`. If the file cannot be opened or processed a null pointer is returned. Geographic coordinates, `val`, by procedure `nad_cvt` are transformed in a forward (`inverse=0`) or inverse (`inverse!=0) manner defined \verb`ctable' and returned function value. If the a conversion cannot be made, the returned results will be set to `HUGE_VAL`. Procedure `nad_free` returns the memory allocated by `nad_init` to the system.

The procedure `dmstor` is a utility to convert DMS type ASCII strings to radians. Usage is identical to the ANSI standard procedure `strtod` where `str` is a pointer to the source data string and if `pstr` is not zero the contents of the pointer it points to will be set to a pointer to the first character in `str` after characters interpreted as part of a DMS formatted word. If an error is detected in conversion, a value of `HUGE_VAL` will be returned.

Procedure `rtodms` converts the radian argument `rad` to a DMS string stored in the location pointed to by `str`. If `pos` is not 0, then `pos` and `neg` are used as sign characters to be suffixed to the converted string, otherwise standard sign prefixing is used. Typically, `pos` is set to `E` or `N` and `W` or `S` for respective conversion of longitude or latitude values of `rad`.

Procedure `set_rtodms` is be used to control aspects of `rtodms` conversion. By default, `rtodms`' conversion truncates trailing zeros and zero seconds or minutes fields and with an assumed precision of $0.001''$. The precision is changed by the value of `frac` which specifies the number of significant fractional seconds digits (default 3) and if `fixed` is non-zero, then fixed field width, with leading zeros are used in the format.

A pointer to a character string is returned by `pj_strerrno` which describes the nature of a non-zero argument value, `errnum`. Positive arguments are operating system errors and negative arguments are errors detected by the **proj** library system. If the argument is 0, a null pointer is returned. The string referenced by the returned pointer should be considered as type `const`.